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Pinning stochastic sampled-data control for exponential synchronization of directed complex dynamical networks with sampled-data communications

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ABSTRACT

This paper is concerned with the exponential synchronization of directed complex dynamical networks (CDNs) with sampled-data communications (SDCs) via pinning stochastic sampled-data control. Different from traditional directed CDNs with determined sampling intervals, multiple stochastic varying sampling intervals with given probabilities are considered in this paper. Compared with some existing control schemes, our control method is more practical because the random sampling intervals always happen in some practical situation. In addition, a Lyapunov–Krasovskii functional (LKF) with some new terms is constructed, which can fully capture the information on stochastic sampling intervals, stochastic input delays, and nonlinear functions. Based on the LKF and Wirtinger's inequality, less conservative synchronization criteria are obtained. Finally, numerical examples are given to illustrate the effectiveness and superiorities of the proposed results.

1. Introduction

In the past few decades, complex dynamical networks (CDNs) have provoked increasing interests, since they have been extensively applied in various fields including biological neural networks, Internet, smart grids, and human social networks [1–9]. One of the most important collective behaviors of CDNs is synchronization, in which the states of all nodes in the network converge towards the same trajectory. Recently, much effort has been devoted to investigate the synchronization of CDNs owing to its potential applications in chemical reactions, secure communication, traffic systems, and biological systems [10–16]. It is, therefore, meaningful in both basic theory and technological practice to study the synchronization of CDNs.

Up to now, various control schemes have been proposed to deal with synchronization for CDNs, such as adaptive control [17], feedback control [18], and impulsive control [19]. All the aforementioned control schemes need to be implemented to each node of CDNs. However, it is practically impossible to implement one controller to each node in a large-scale network [20]. Pinning control, by which only a small fraction of the nodes need to be pinned, has attracted much attention and

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many interesting results have been reported in the literature [20–23]. Hence, the investigation of synchronization for CDNs via pinning control is of great importance.

With the rapid developments of high-performance computers and communication networks, a designed continuous-time feedback controller is usually implemented in a digital form. The digital form controller shows more advantages, such as low cost consumption, high reliability, and easy installation. Thus, the sampled-data controller, which uses only the sampled information of the system at its instants, has been becoming a hot research topic [24–35]. For example, in [29], by designing an appropriate sampled-data controller, the stabilization problem for sampled-data neural-network-based systems under variable sampling has been investigated. In [30], the stability and stabilization problems of a class of Markovian chaotic systems have been studied via fuzzy sampled-data control. In [35], based on a T–S fuzzy model, the stabilization has been considered for chaotic systems under sampled-data control and state quantization.

The investigation of sampled-data systems has fallen into mainly three approaches, that is, discrete-time approach [36], impulsive model approach [37], and input delay approach [38]. The most popular approach is input delay approach, which is developed in the sense of continuous-time perspective. Based on input delay approach, many interesting results have been reported on sampled-data synchronization of CDNs [39–42]. However, in [39–42], the information transmission among the nodes is assumed to be continuous, which implies that the coupling law of the network is executed by analog signal. Moreover, in practice, the communication topology among some networks may be directed. Thus, it is of great importance to study the synchronization of directed CDNs with sampled-data communications (SDCs). Recently, such topic has been discussed in [20,21]. In [20], the synchronization problem for directed CDNs with SDCs has been investigated by presenting a algorithm to determine the nodes to be pinned. In [21], based on Writinger-based integral inequality, improved results have been established for directed CDNs with SDCs. It is well-known that if we utilize more information of the systems, then the results might be improved. However, in [20,21], the derived synchronization conditions are conservative to some extent, since the information on neuron activation functions has been ignored in the constructed LKFs. Thus, how to create a new LKF to fully capture the information on neuron activation functions for synchronization of directed CDNs with SDCs is the first motivation of this note.

As well known, it is essential to select proper sampling intervals for sampled-data systems to guarantee a desired performance. Most of the existing results [20,21,39–42] focus more on sampling signal with a constant rate. In practice, due to the uncertain interferences induced by environmental and artificial factors, the sampling intervals may vary with the network condition [43]. Hence, it is necessary to consider stochastic varying sampling intervals for synchronization of directed CDNs with SDCs. To our best knowledge, few results have been reported on the exponential synchronization of directed CDNs with SDCs via pinning stochastic sampled-data control. This is the second motivation of this work.

Motivated by the aforementioned discussions, we intend to study the exponential synchronization problem of directed CDNs with SDCs via pinning stochastic sampled-data control. The main contributions are summarized below.

(1) Pinning sampled-data control with stochastic varying sampling intervals is first designed for exponential synchronization of directed CDNs with SDCs. Compared with the existing pinning sampled-data control schemes in [20,21], the control scheme here is more applicable, since stochastic varying sampling intervals always happen in some practical situation.

(2) A LKF with some new terms is presented for directed CDNs with SDCs. Specially, three (t_k, t_{k+1})-dependent terms $V_i(t)$ $(i = 2, 3, 4)$ can make full use of the information on the stochastic sampling intervals and stochastic input delays. The term $V_5(t)$ can take full advantage of the information on nonlinear functions. Compared with the results in [20,21], our derived synchronization criterion is less conservative.

Notations: Throughout this paper, $\Re^n$ and $\Re^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. I_n, 0_n, and $0_{n,m}$ stand for $n\times n$ identity matrix, $n\times n$ and $n\times m$ zero matrices, respectively. The superscript T means the transpose of a matrix. For real symmetric matrices X and Y, the notation $X > Y$ means that the matrix $X - Y$ is positive definite. $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a real symmetric matrix. diag$\{\cdots\}$ and col$\{\cdots\}$ stand for a block-diagonal matrix and a column vector, respectively. Sym$\{X\} = X + X^T$. The symmetric term in a matrix is denoted by $*$. $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix. The symbol $\otimes$ means the Kronecker product. $\mathscr{E}\{x\}$ means the mathematical expectation.

2. Problem description and preliminaries

Some algebra graph theories are presented for the comprehension of the following results. Let $\mathcal{G}(\mathcal{V}, \varepsilon, \mathcal{A})$ be a weighted directed graph of order N, where $\mathcal{V} = \{1, 2, \ldots, N\}$ refers to the set of nodes, $\varepsilon \subseteq \mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{A} = [a_{ij}]_{N\times N}$ is the weighted adjacent matrix. For each node i, the in-degree is defined as $\mathcal{D}(i) = \sum_{j=1}^{N} a_{ij}$. Then, the extensively used Laplacian matrix can be given by $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D}$ is the diagonal matrix with $\mathcal{D}(i)$ as the ith diagonal element. In the sequel, denote $\mathcal{G}(\mathcal{V}, \varepsilon, \mathcal{A})$ by $\mathcal{G}(\mathcal{A})$ if no confusion will occur. A directed graph $\mathcal{G}(\mathcal{A})$ is called strongly connected if there exists at least one directed path between any pair of distinct nodes i and j [44]. $\mathcal{G}(\mathcal{A})$ contains a directed spanning tree if there exists a node r, called a root, such that there exists a directed path from this node to every other node [44].

Consider the following CDN consisting of N nodes with sampled-data based diffusive couplings:

$$\dot{x}_i(t) = f(x_i(t), t) + c\sum_{j=1}^{N} a_{ij}(x_j(t_k) - x_i(t_k)) + u_i(t), \ \ t_k \le t < t_{k+1}, \ \ i = 1, 2, \ldots, N, \tag{1}$$