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Stochastic fractional evolution equations with fractional brownian motion and infinite delay☆

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ABSTRACT

In this paper, we consider a class of stochastic fractional evolution equations with infinite delay and a fractional Brownian motion in a Hilbert space. By the stochastic analysis technique, we establish the existence and uniqueness of mild solutions for these equations under non-Lipschitz condition with Lipschitz conditions being considered as a special case. An example is provided to illustrate the theory.

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1. Introduction

In recent years, fractional calculus and fractional differential equations have attracted the attention of many researchers due to their important applications to problems in mathematical physics, chemistry, biology and engineering. Many results on existence and stability of solutions to various type of fractional differential equations has been obtained. For more details on this topic, one can refer to [10,14,35].

The deterministic models often fluctuate due to noise. Systems are often subjected to random perturbations. Stochastic (partial) differential equations have been investigated by many authors due to playing a very important role in formulation and analysis of many phenomena in economic and finance, in physics, mechanics, electric and control engineering, etc. There is much current interest in studying qualitative properties for SPDEs (see, e.g., Da Prato and Zabczyk [12], Liu [17], Wei and Wang [34], Luo and Liu [19], Jahanipur [13], and references therein). Some authors have consider fractional stochastic partial equations, we refer to Cui and Yan [8], Benchaabane and Sakthivel [2], Sakthivel et al. [29,30,32], Röckner et al. [27], the perturbed terms of these fractional equations are Wiener processes.

One solution for many SDEs is a semimartingale as well a Markov process. However, many objects in real world are not always such processes since they have long-range aftereffects. Since the work of Mandelbrot and Van Ness [21], there is an increasing interest in stochastic models based on the fractional Brownian motion. A fractional Brownian motion (fBm) of Hurst parameter $H\in(0,1)$ is a centered Gaussian process $B^H=\{B^H(t), t\geq 0\}$ with the covariance function

$$R_H(t,s)=\mathbb{E}(B^H(t)B^H(s))=\frac{1}{2}(t^{2H}+s^{2H}-|t-s|^{2H}).$$

When $H=1/2$ the fBm becomes the standard Brownian motion, and the fBm B^H neither is a semimartingale nor a Markov process if $H\neq 1/2$. However, the fBm B^H, $H>1/2$ is a long-memory process and presents an aggregation behavior. The

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long-memory property make fBm as a potential candidate to model noise in mathematical finance (see [7,24]); in biology (see [5,11]); in communication networks (see, for instance [33]); the analysis of global temperature anomaly [26] and electricity markets [28] etc.

Recently, stochastic partial functional differential equations driven by fractional Brownian motion have attracted the interest of many researchers. For example, under the global Lipschitz condition, Caraballo et al. [6] showed the existence, uniqueness and stability of mild solutions for SPDEs with finite delays driven by a fBm; under the global Lipschitz condition, Boufoussi and Hajji [3] considered the existence and uniqueness of mild solutions to neutral SPDEs with finite delays driven by a fBm; Boufoussi et at. [4] obtained the existence and uniqueness result of mild solution to a class of of time-dependent stochastic functional differential equations driven by a fBm; Ren et at. [25] proved the existence and uniqueness of the mild solution for a class of time-dependent stochastic evolution equations with finite delay driven by a standard cylindrical Wiener process and an independent cylindrical fractional Brownian motion. To the best of our knowledge, there is few result on stochastic fractional differential equations driven by fBm. One can only see [1,15,18] and the references therein. In [15], Li established the existence and uniqueness of mild solutions to stochastic fractional evolution equations with finite delay driven by a fractional Brownian motion.

On the other hand, in many areas of science, there has been an increasing interest in the investigation of the systems incorporating memory or aftereffect, i.e., there is the effect of infinite delay on state equations. Therefore, there is a real need to discuss stochastic evolution systems with infinite delay. There exist only a few results on SPDEs with infinite delay. One can see [8,9,22,31] and the references therein. However, as far as we known, no work has been reported in the present literature regarding on stochastic fractional differential equations with infinite delay and a fractional Brownian motion. To close the gap, we will make the first attempt to study such problem in this paper. We aim to derive the existence and uniqueness of mild solutions to a class of stochastic fractional differential equations with infinite delay and a fractional Brownian motion under some local conditions.

The rest of this paper is organized as follows. In Section 2, we introduce some necessary notations and preliminaries. In Section 3, the existence and uniqueness of mild solutions are discussed. An example is presented in Section 4 to illustrate the theory.

2. Preliminaries

In this section we collect some notions, conceptions and lemmas on Wiener integrals with respect to an infinite dimensional fractional Brownian motion and recall some basic results which will be used throughout the whole of this paper.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a filtered complete probability space satisfying the usual condition, which means that the filtration is a right continuous increasing family and $\mathcal{F}_0$ contains all P-null sets.

Now we aim at introducing the Wiener integral with respect to the one-dimensional fBm B^H. Consider a time interval $[0, T]$ with arbitrary fixed horizon T and let $\{B^H(t), t\in[0, T]\}$ be the one-dimensional fractional Brownian motion with Hurst parameter $H\in(1/2, 1)$. This means B^H has the following Wiener integral representation:

$$B^H(t) = \int_0^t K_H(t,s)dB(s),$$

where $B = \{B(t) : t \in [0, T]\}$ is a standard Brownian motion, and $K_H(t, s)$ is the kernel given by

$$K_H(t,s) = c_H s^{\frac{1}{2}-H}\int_s^t (u-s)^{H-\frac{3}{2}}u^{H-\frac{1}{2}}du$$

for $t > s$, where $c_H = \sqrt{\frac{H(2H-1)}{\beta(2-2H, H-\frac{1}{2})}}$. We put $K_H(t,s) = 0$ if $t \leq s$.

We will denote by $\mathcal{H}$ the reproducing kernel Hilbert space of the fBm. In fact $\mathcal{H}$ is the closure of the linear space of indicator functions $\{I_{[0,\,t]}, t\in[0, T]\}$ with respect to the scalar product

$$\langle I_{[0,t]}, I_{[0,s]}\rangle_{\mathcal{H}} = R_H(t,s).$$

The mapping $I_{[0,t]} \to B^H(t)$ can be extended to an isometry between $\mathcal{H}$ and the first Wiener chaos and we will denote by $B^H(\varphi)$ the image of φ by the such isometry.

We recall that for $\psi, \varphi \in \mathcal{H}$ their scalar product in $\mathcal{H}$ is given by

$$\langle \psi, \varphi\rangle_{\mathcal{H}} = H(2H-1)\int_0^T\int_0^T \psi(s)\varphi(t)|t-s|^{2H-2}dsdt.$$

Let us consider the operator K_H^* from $\mathcal{H}$ to $L^2([0, T])$ defined by

$$(K_H^*\varphi)(s) = \int_s^T \varphi(r)\frac{\partial K}{\partial r}(r,s)dr.$$

We refer to [23] for the proof of the fact that K_H^* is an isometry between $\mathcal{H}$ and $L^2([0, T])$. Moreover, for any $\varphi \in \mathcal{H}$, we have

$$B^H(\varphi) = \int_0^T (K_H^*\varphi)(t)dB(t).$$

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