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The g-extra connectivity and diagnosability of crossed cubes

Shiying Wang*, Xiaolei Ma

School of Mathematics and Information Science, Henan Normal University, Xinxiang, Henan 453007, PR China

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ABSTRACT

Connectivity and diagnosability are two important parameters for the fault tolerant of an interconnection network G. In 1996, Fàbrega and Fiol proposed the g-extra connectivity of G. In 2016, Zhang et al. proposed the g-extra diagnosability of G that requires every component of $G-S$ has at least $(g+1)$ vertices. The g-extra connectivity of G is necessary for g-extra diagnosability of G. In this paper, we show that the g-extra connectivity of the crossed cube CQ_n is $n(g+1)-\frac{1}{2}g(g+3)$ for $n\geq 5$, $0\leq g\leq \lfloor\frac{n}{2}\rfloor$ and the g-extra diagnosability of CQ_n is $(n-\frac{1}{2}g)(g+1)$ under the PMC model for $n\geq 5$, $0\leq g\leq \lfloor\frac{n}{2}\rfloor$ and the MM* model for $n\geq 7$, $0\leq g\leq \lfloor\frac{n}{2}\rfloor$.

1. Introduction

The processors of a multiprocessor system are connected according to a given interconnection network. However, the failures of the processor are unavoidable. How to find the fault processors in time and accurately becomes the main problem to maintain the stability of the system. The process of identifying the faulty processors is called the diagnosis of the system. A system G is said to be t-diagnosability if all faulty processors can be identified without replacement, provided that the number of faults presented does not exceed t. The diagnosability $t(G)$ of G is the maximum value of t such that G is t-diagnosable. Dahbura and Masson [3] proposed an algorithm with time complex $O(n^{2.5})$, which can effectively identify the set of faulty processors. There are two well-known diagnostic models, one of which is the PMC model, proposed by Preparata et al. [13] and the other is the MM model, introduced by Maeng and Malek [12]. In the PMC model, any two adjacent processors can test each other. In the MM model, a processor sends the same task to its two neighboring processors and compares the feedback results. Sengupta and Dahbura [15] proposed proposed a special case of the MM model, called the MM* model, in which each processor must test its any pair of adjacent processors. Recently, there are some new measures of fault tolerance have been proposed. In 2016, Zhang et al. [23] proposed the g-extra diagnosability of the system, which restrains that every fault-free component has at least $(g+1)$ fault-free processors. And they studied the g-extra diagnosability of the n-dimensional hypercube under the PMC model and the MM* model. There are some related articles in [16,17,20,23].

In 1996, the g-extra connectivity $\widetilde{\kappa}^{(g)}(G)$ of the interconnection network G was introduced by Fàbrega and Fiol [6]. The g-extra connectivity $\widetilde{\kappa}^{(g)}(G)$ of G has been widely studied [2,6–9,11,14,18,19,24–26]. We mainly study the g-extra diagnosability of the crossed cube CQ_n [4] in this paper, but the g-extra connectivity of CQ_n is necessary for the g-extra diagnosability of CQ_n. The hypercube is a well-known interconnection model for multiprocessor systems. A various of important variants of the hypercube have been proposed, such as the crossed cube, twisted cube and möbius cube. The main difference is

* Corresponding author.
E-mail addresses: wangshiying@htu.edu.cn, shiying@sxu.edu.cn (S. Wang).

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that each of these cubes has various perfect matching between its subcubes. Therefore, they are called the hypercube-like networks (HL-networks) or bijective connection networks (BC-networks), denoted by $\mathbb{L}_n$. Let $X_n \in \mathbb{L}_n$. In [21,25], Yang and Zhou proved that the g-extra connectivity of X_n is more than or equal to $n(g+1) - \frac{1}{2}g(g+3)$ for $n \geq 5$, $0 \leq g \leq n-3$. They investigated that the g-extra connectivity of a varietal hypercube VQ_n is $n(g+1) - \frac{1}{2}g(g+3)$ for $n = 3s+t$, $0 \leq g \leq n-s$ with $s \geq 3$ and $0 \leq t \leq 2$. In addition, they presented a subclass of n-dimension HL-networks with g-extra connectivity greater than $n(g+1) - \frac{1}{2}g(g+3)$. In this paper, we will study the n-dimensional crossed cube CQ_n [4], which is an n-regular graph and has 2^n vertices. We show that the g-extra connectivity of the crossed cube CQ_n is $n(g+1) - \frac{1}{2}g(g+3)$ for $n \geq 5$, $0 \leq g \leq \lfloor \frac{n}{2} \rfloor$ and the g-extra diagnosability of CQ_n is $(n - \frac{1}{2}g)(g+1)$ under the PMC model for $n \geq 5$, $0 \leq g \leq \lfloor \frac{n}{2} \rfloor$ and the MM* model for $n \geq 7$, $0 \leq g \leq \lfloor \frac{n}{2} \rfloor$.

2. Preliminaries

2.1. Notations

A multiprocessor system is modeled as an undirected simple graph $G = (V, E)$, whose vertices (nodes) V represent processors and edges (links) E represent communication links. For $V' \subseteq V$, suppose $V' \neq \emptyset$. The induced subgraph by V' in G, denoted by $G[V']$, is a graph, whose vertex set is V' and whose edge set consists of all the edges of G with both ends in V'. Let F_1 and F_2 be two distinct subsets of V, and let the symmetric difference $F_1 \triangle F_2 = (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$. For any vertex v, we define the neighborhood $N_G(v)$ of v in G to be the set of vertices adjacent to v. u is called a neighbor vertex (a neighbor) of v for $u \in N_G(v)$. Let $S \subseteq V(G)$. The set $\cup_{v \in S} N_G(v) \setminus S$ is denoted by $N_G(S)$. The degree $d_G(v)$ of a vertex v in G is the number of edges incident with v. For neighborhoods and degrees, we will usually omit the subscript for the graph when no confusion arises. A graph G is said to be k-regular if $d_G(v) = k$ for any vertex $v \in V$. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left. A bipartite graph is one whose vertex set can be partitioned into two subsets X and Y; so that each edge has one end in X and one end in Y; such a partition (X, Y) is called a bipartition of the graph. A complete bipartite graph is a simple bipartite graph with bipartition (X, Y) in which each vertex of X is joined to each vertex of Y; if $|X| = m$ and $|Y| = n$, such a graph is denoted by $K_{m,n}$. For graph-theoretical terminology and notation not defined here we follow [1].

Definition 2.1. For a graph $G = (V, E)$, a faulty set $F \subseteq V$ is called a g-extra faulty set if every component of $G - F$ has at least $(g+1)$ vertices.

Definition 2.2. A g-extra cut of a graph G is a g-extra faulty set F such that $G - F$ is disconnected. G is said to be g-extra connected if G has a g-extra cut. Let G be g-extra connected. The minimum cardinality of g-extra cuts is said to be the g-extra connectivity of G, denoted by $\tilde{\kappa}^{(g)}(G)$.

Definition 2.3. A g-extra connected graph G is super g-extra connected if every minimum g-extra cut F of G isolates one connected subgraph of order $g+1$. In addition, if $G - F$ has two components, one of which is the connected subgraph of order $g+1$, then G is tightly $|F|$ super g-extra connected.

Theorem 2.4 [3,16,22]. *A system $G = (V, E)$ is g-extra t-diagnosable under the PMC model if and only if there is an edge $uv \in E$ with $u \in V \setminus (F_1 \cup F_2)$ and $v \in F_1 \triangle F_2$ for each distinct pair of g-extra faulty subsets F_1 and F_2 of $V(CQ_n)$ with $|F_1| \leq t$ and $|F_2| \leq t$.*

Theorem 2.5 [15,16,22]. *A system $G = (V, E)$ is g-extra t-diagnosable under the MM* model if and only if each distinct pair of g-extra faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$ satisfies one of the following conditions. (1) There exist two vertices $u, w \in V \setminus (F_1 \cup F_2)$ and there exists a vertex $v \in F_1 \triangle F_2$ such that $uw, vw \in E$. (2) There exist two vertices $u, v \in F_1 \setminus F_2$ and there exists a vertex $w \in V \setminus (F_1 \cup F_2)$ such that $uw, vw \in E$. (3) There exist two vertices $u, v \in F_2 \setminus F_1$ and there exists a vertex $w \in V \setminus (F_1 \cup F_2)$ such that $uw, vw \in E$.*

The g-extra diagnosability of G, denoted by $\tilde{t}_g(G)$, is the maximum value of t such that G is g-extra t-diagnosable.

2.2. Crossed cubes

Definition 2.6. Let $R = \{(00, 00), (10, 10), (01, 11), (11, 01)\}$. Two digit binary strings $u = u_1u_0$ and $v = v_1v_0$ are pair related, denoted as $u \sim v$, if and only if $(u, v) \in R$.

Definition 2.7. The vertex set of a crossed cube CQ_n is $\{v_{n-1}v_{n-2}\ldots v_0 : 0 \leq i \leq n-1, v_i \in \{0, 1\}\}$. Two vertices $u = u_{n-1}u_{n-2}\ldots u_0$ and $v = v_{n-1}v_{n-2}\ldots v_0$ are adjacent if and only if one of the following conditions is satisfied (see Fig. 1).

1. There exists an integer l $(1 \leq l \leq n-1)$ such that
   (1) $u_{n-1}u_{n-2}\ldots u_l = v_{n-1}v_{n-2}\ldots v_l$;
   (2) $u_{l-1} \neq v_{l-1}$;
   (3) if l is even, $u_{l-2} = v_{l-2}$;
   (4) $u_{2i+1}u_{2i} \sim v_{2i+1}v_{2i}$, for $0 \leq i < \lfloor \frac{l-1}{2} \rfloor$.