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Reduced-order observer design for a class of generalized Lipschitz nonlinear systems with time-varying delay ☆

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ABSTRACT

This paper investigates the H_∞ reduced-order observer design problem for a class of nonlinear systems with interval time-varying delay which satisfies the quadratically inner-bounded condition and encompasses the family of Lipschitz systems. A novel reduced-order observer design methodology for nonlinear systems is proposed. By utilizing a newly extended reciprocal convexity inequality, free-weighting matrix technique, and quadratically inner-bounded condition, the less conservative existence conditions of the proposed nonlinear H_∞ observer are derived. The new sufficient conditions in terms of linear matrix inequalities (LMIs) guarantee asymptotic stability of the estimation error dynamics with a prescribed performance γ. Two numerical examples are given to illustrate the effectiveness of the proposed approach.

1. Introduction

Observer design problem for nonlinear systems has been an important research topic in control theory [1,2,6,7]. It is well known that when not all the internal states of the dynamic system are available, a state observer is usually required for implementation of the control design. Observer design is also significantly employed in many engineering applications, such as process monitoring, fault detection and isolation, neural network [1], and chaos synchronization-based secure communication [2]. The nonlinear observer design is relatively difficult compared with the observer design theory for linear systems. Generally speaking, there is no general approach for the nonlinear observer design problem although most physical and engineering systems are inherently nonlinear.

Most of the existing nonlinear observer design methods focus on certain special class of nonlinearities. The Lipschitz system is such a popular class of nonlinear systems investigated by the researchers in the past decades [7,8,19]. Many of the physical system models satisfy the Lipschitz condition globally or at least locally. The drawback of existing observer methods for Lipschitz system is that they fail to deal with large values of Lipschitz constant. Hu [9] firstly introduced the one-sided Lipschitz condition of nonlinear systems to overcome this limitation. Recently, Abbaszadeh and Marquez [11] extended Hu's

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one-sided Lipschitz condition and meanwhile introduced another new concept named quadratically inner-bounded condition to make the design tractable. It is known that both the one-sided Lipschitz systems [10,12–14] and quadratically inner-bounded systems encompass the traditional Lipschitz systems and then these kinds of systems exhibit inherent advantages with respect to conservativeness. Thereafter, observer design for nonlinear systems satisfying one-sided Lipschitz condition and quadratically inner-bounded condition simultaneously are commonly investigated in [15,17,18,20–29]. Nonlinear H_∞ observer for one-sided Lipschitz nonlinear systems was proposed in [15]. The observer design for discrete-time one-sided Lipschitz nonlinear systems was investigated in [18,24]. Observer design for one-sided Lipschitz descriptor systems was presented in [23]. The observer-based control problem for one-sided Lipschitz systems was studied in [21]and [22]. Robust observer design for uncertain one-sided Lipschitz systems with disturbances was researched in [28].

Indeed, the nonlinear systems considered in the aforementioned reference are only a subset of quadratically inner-bounded nonlinear systems, which are also one-sided Lipschitz. In [16], the authors presented the improved exponential observer design for nonlinear systems satisfying one-sided Lipschitz condition only. A natural question will be similarly put forward that can we consider the quadratically inner-bounded condition only? This will make it possible to reduce the conservatism and complexity of the design since checking these two conditions is not so easy for many nonlinear functions. In addition, time delay is also a challenging problem which may cause unsatisfactory performance and instability of dynamical systems [4]. Nonlinear observer design for time-delay nonlinear system is more complicated since extra effort should be made to deal with the time-delay issue. Nonlinear reduced-order observers were proposed to one-sided Lipschitz discrete-time and continuous-time systems with time-varying delay quite recently in [25–27]. LMI conditions for the observer synthesis problems of one-sided Lipschitz nonlinear systems with interval time-varying delay and uncertainties were presented in [20]. In a word, the problem of designing observers for time-delay nonlinear systems is not fully investigated. This motivates the present work of this paper. In this paper, we stress on reduced-order observer design for a general class of delayed nonlinear systems that satisfies the quadratically inner-bounded condition only. The main contributions of the paper are summarized below:

(1) A novel H_∞ reduced-order observer design methodology for a class of nonlinear systems with time-varying delay is proposed.

(2) Unlike some existing works dealing with nonlinear systems satisfying one-sided Lipschitz condition and the quadratically inner-bounded condition simultaneously, the present work removes the constraint of one-sided Lipschitz condition and considers quadratically inner-bounded condition only, which will significantly reduce the conservatism and extend the observer design to a larger class of nonlinear systems.

(3) Some new and less conservative synthesis conditions of observer design which can be tackled by Matlab tool box directly are presented for nonlinear systems with interval time-varying delay by employing an extended reciprocal convexity inequality quite recently proposed by Zhang et al. [3].

This paper is organized as follows. Section 2 gives the problem formulation and preliminaries. Section 3 presents the main results. Simulation results showing the efficiency and high performance of proposed observer design approach are provided in Section 4. Section 5 draws conclusions of the study.

Notation: The notation throughout this paper is standard. A^T denotes the transpose of matrix A. I and 0, respectively, are the identity matrix and zero matrix with appropriate dimensions. The symbol * represents symmetric terms in a symmetric matrix and for any $A \in \mathbb{R}^{n\times n}$, we define $Sym\{A\} = A + A^T$. $\|x\|$ is the Euclidean norm of vector x. $\langle x, y\rangle = x^T y$ is the inner product of vectors x, y. $\|x\|_{\mathcal{L}_2} = (\int_0^\infty \|x\|^2 dt)^{\frac{1}{2}}$ is the the $\mathcal{L}_2$ norm of vector x, where $\mathcal{L}_2$ is the set defined by $\mathcal{L}_2 = \{x : \|x\|_{\mathcal{L}_2} < +\infty\}$.

2. Problem formulation and preliminaries

We consider the following nonlinear system with time-varying delay

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + A_d x(t-d(t)) + B_1 f_1(F_1 x(t), u(t)) + B_2 f_2(F_2 x(t-d(t)), u(t-d(t))) + Bu(t) + Gw(t) \\
y(t) &= Cx(t) \\
x(t) &= \phi(t), \ \ t \in [-h_2, 0],
\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input,$y(t) \in \mathbb{R}^p$ is the measurement output, $w(t) \in \mathbb{R}^q$ is the disturbance input ; $A, A_d \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, B_1 \in \mathbb{R}^{n\times m_1}, B_2 \in \mathbb{R}^{n\times m_2}, F_1 \in \mathbb{R}^{m_1\times n}, F_2 \in \mathbb{R}^{m_2\times n}, G \in \mathbb{R}^{n\times q}, C \in \mathbb{R}^{p\times n}$ are known real constant matrices with C being of full row rank; $\phi(t)$ is the initial condition; The delay $d(t)$ is a known time-varying delay of the system which satisfies

$$0 \le h_1 \le d(t) \le h_2, \quad \mu_1 \le \dot{d}(t) \le \mu_2. \tag{2}$$

where h_1, h_2, μ_1 and μ_2 are known constants.

Assume that system (1) has a unique solution. We introduce the following two assumptions which are commonly used in the recent literature for nonlinear observer design.

Assumption 1. The nonlinear functions $f_i(F_i x, u) : \mathbb{R}^{m_i} \times \mathbb{R}^m \to \mathbb{R}^{m_i} (i = 1, 2)$ are one-sided Lipschitz [11], i.e.,

$$\langle f_i(F_i x, u) - f_i(F_i\hat{x}, u), F_i(x-\hat{x})\rangle \le \rho_i \|F_i(x-\hat{x})\|^2, \forall x, \hat{x} \in \mathbb{R}^n, u(t) \in \mathbb{R}^m \tag{3}$$