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Dynamics of a stochastic delay competitive model with harvesting and Markovian switching

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ABSTRACT

This work is concerned with a two-species stochastic state-switching competitive population model with distributed delays and harvesting. First, necessary and sufficient criteria for the existence of a unique ergodic stationary distribution of the system are established. Then necessary and sufficient criteria for the existence of the optimal harvesting policy are given, and the explicit expression of the optimal harvesting policy is obtained. Finally, some effects of the state-switching on the persistence, extinction and optimal harvesting strategy of the system are discussed with the help of several simulations.

1. Introduction

Establishing ecologically and economically reasonable optimal harvesting policies is a key problem in mathematical biology [1]. In addition, the real world is filled with random perturbations, and in many cases, these random perturbations should not be ignored. Therefore in recent years, many researchers have drawn their attention in the optimal harvesting problems of population models with random perturbations (see, e.g., [2–14]). Beddington and May [2] have obtained the optimal harvesting policy (OHP) for a stochastic logistic population model by solving the corresponding Fokker–Planck equation. Using the same method in [2], the OHP of a stochastic Gilpin–Ayala model has been established by Li and Wang [3]. However, for most stochastic population models, it is impossible to solve the corresponding Fokker–Planck equations explicitly. To solve this problem, Zou et al. [6] have proposed a so-called “ergodic approach”, and have used this approach to establish the OHP for a singe-species Gompertz model with random perturbations. From then on, the “ergodic approach” has been applied to explore the OHP of many population models in random environments, for example, logistic models [7,9,10], competitive models [11,12] and predator-prey models [13,14].

Most stochastic population models with harvesting are driven by the Brownian motion. However, some common stochastic perturbations in the real world can not be described by the Brownian motion, for example, the telephone noise. Telephone noise is a switching between two or more environmental states [15–18]. The switching between different states is memoryless, and the time between two switchings is exponentially distributed [16–18]. Many scholars (see, e.g., [16–19]) have claimed that the growth rates of the species are often subject to the telephone noise, an example is that in monsoon forest, the growth rates of some populations often change due to rainfall [20]. In addition, in the real world, competition and delay are usual phenomena. It is therefore of great significance to consider delay competitive models with harvesting and telephone noises. However, as far as we know, a very little amount of work has been done with the harvesting delay

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competitive models perturbed by the telephone noise, and the impacts of the telephone noise on the optimal harvesting of the delay competitive models are still unknown. Motivated by these, in this paper we study the dynamics of a delay stochastic competitive model with harvesting and state-switching.

The rest of the paper is arranged as follows. In Section 2, our model is formulated by using a continuous-time Markov chain to describe the telephone noise. In Section 3, the critical values between the existence of a unique ergodic stationary distribution (UESD) and the collapse of the model are obtained. In Section 4, sufficient and necessary criteria for the existence of OHP are established, and the explicit expressions of the optimal harvesting effort (OHE) and the maximum of sustainable yield (MSY) are gained. In Section 5, some interesting and important influences of the telephone noise on the the existence of a UESD, the extinction and the optimal harvesting strategy of the model are discussed and revealed with the help of several examples and numerical simulations. Section 6 ends this paper with some conclusions and discussions.

2. Model formulation

First of all, let us consider the following deterministic two-species competitive model with time delays ([21]) :

$$\begin{cases} \dfrac{dx_1(t)}{dt} = x_1(t)\Big[b_1 - a_{11}x_1(t) - a_{12}\displaystyle\int_{-\tau_1}^{0} x_2(t+\eta)d\xi_1(\eta)\Big], \\ \dfrac{dx_2(t)}{dt} = x_2(t)\Big[b_2 - a_{21}\displaystyle\int_{-\tau_2}^{0} x_1(t+\eta)d\xi_2(\eta) - a_{22}x_2(t)\Big], \end{cases} \tag{1}$$

with initial value

$$x(\eta) \in \mathscr{C}, \tag{2}$$

where $x_i(t)$ stands for the size of the species i at time t, $i = 1, 2$; $b_i > 0$ represents the growth rate of the species i; $a_{ii} > 0$ denotes the intra-specific competition of the ith species; $a_{ij} > 0$ $(j \neq i)$ stands for the interspecific competition; $\xi_i(\cdot)$ is a probability measure on $[-\tau, 0]$ with $\int_{-\tau_i}^{0} d\xi_i(\eta) = 1$, $i = 1, 2$, $\tau = \max\{\tau_1, \tau_2\}$, $\mathscr{C}$ represents the family of all continuous functions from $[-\tau, 0]$ to $\mathbb{R}_+^2 := \{a \in \mathbb{R}^2 | a_1 > 0, a_2 > 0\}$.

When the fishing pressure to species i is added at the strength of H_i ([1–3,9,14,19]), model (1) becomes

$$\begin{cases} \dfrac{dx_1(t)}{dt} = x_1(t)\Big[b_1 - H_1 - a_{11}x_1(t) - a_{12}\displaystyle\int_{-\tau_1}^{0} x_2(t+\eta)d\xi_1(\eta)\Big], \\ \dfrac{dx_2(t)}{dt} = x_2(t)\Big[b_2 - H_2 - a_{21}\displaystyle\int_{-\tau_2}^{0} x_1(t+\eta)d\xi_2(\eta) - a_{22}x_2(t)\Big], \end{cases}$$

where H_i is the harvesting effort of species i.

Now let us formulate the stochastic model. To begin with, we consider the perturbations of the telephone noise on the growth rates of the species. Following the procedure given in [16–18,20,22,23], we use a continuous-time Markov chain $\{\beta(t), t \geq 0\}$ with finite-state space $\Pi = \{1, \dots, m^*\}$ to model the telephone noise, then we obtain the following model

$$\begin{cases} \dfrac{dx_1(t)}{dt} = x_1(t)\Big[b_1(\beta(t)) - H_1 - a_{11}x_1(t) - a_{12}\displaystyle\int_{-\tau_1}^{0} x_2(t+\eta)d\xi_1(\eta)\Big], \\ \dfrac{dx_2(t)}{dt} = x_2(t)\Big[b_2(\beta(t)) - H_2 - a_{21}\displaystyle\int_{-\tau_2}^{0} x_1(t+\eta)d\xi_2(\eta) - a_{22}x_2(t)\Big]. \end{cases} \tag{3}$$

Secondly, let us incorporate the white noise into the above system. Several authors (see, e.g., [2]) have pointed out that the growth rates are the most sensitive parameters and are often affected by the white noise. In practice, one usually use an average value plus an error term to estimate the growth rate b_i [3,17,18,24–26]. In view of the central limit theorem, the error term is normally distributed. Therefore (see, e.g., [2,3,18])

$$b_i(\cdot) \to b_i(\cdot) + \sigma_i(\cdot)\dot{W}_i(t), \quad i = 1, 2.$$

Here the average growth rate is still denoted as b_i for simplicity; $\dot{W}_i(t)$ represents the white noise, i.e., $\{W(t)\}_{t\geq 0} = \{W_1(t), W_2(t)\}_{t\geq 0}$ is a two-dimensional Wiener process defined on a complete probability space $(\Omega, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$; σ_i^2 stands for the intensity of the white noise. Hence model (3) becomes:

$$\begin{cases} dx_1(t) = x_1(t)\Big[b_1(\beta(t)) - H_1 - a_{11}x_1(t) - a_{12}\displaystyle\int_{-\tau_1}^{0} x_2(t+\eta)d\xi_1(\eta)\Big]dt + \sigma_1(\beta(t))x_1(t)dW_1(t), \\ dx_2(t) = x_2(t)\Big[b_2(\beta(t)) - H_2 - a_{21}\displaystyle\int_{-\tau_2}^{0} x_1(t+\eta)d\xi_2(\eta) - a_{22}x_2(t)\Big]dt + \sigma_2(\beta(t))x_2(t)dW_2(t), \end{cases} \tag{4}$$