Applied Mathematics and Computation 336 (2018) 215–230

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A new approach for space-time fractional partial differential equations by residual power series method

Mine Aylin Bayrak*, Ali Demir

Department of Mathematics, Kocaeli University, Kocaeli, Turkey

ARTICLE INFO

MSC:
35K35
26A33
35F25

Keywords:
Residual power series method
Space-time fractional partial differential equations
Caputo derivative

ABSTRACT

In this paper, the approximate analytic solution of any order space-time fractional differential equations is constructed by means of semi-analytical method, named as residual power series method (RPSM). The first step is to reduce space-time fractional differential equation to either a space fractional differential equations or a time fractional differential equations before applying RSPM. The main step is to obtain fractional power series solutions by RSPM. At the final step, it is shown that RPSM is very efficacious, plain and powerful for obtaining the solution of any-order space-time fractional differential equations in the form of fractional power series by illustrative examples.

1. Introduction

The concept of fractional calculus has been gaining considerable attention of many scientists since it has many applications in various fields such as fluid flow, regular variation in thermodynamics, aerodynamics, electrochemistry of corrosion, biology, optics and signal processing and so on [1–4]. Since it is a very efficacious, plain and powerful tool for the investigation of engineering and scientific phenomena, it has been extensively investigated in the last few decades. New definitions of fractional derivatives with singular and nonsingular kernels are constructed to solve fractional differential equations [5–8]. There are various methods for the solution of fractional differential equations in literature [9–17].

Many problems in mathematics and physics has to be evaluated in time and space therefore for the modelling of this kind of problems, fractional partial differential equations (FPDEs) are required. For the solutions of the space-time FPDEs, existence and uniqueness of the weak solution was studied by using existing theory for elliptic problems [18]. RPSM (proposed by the Jordan mathematician Abu Arqub [19]) is an efficient analytical method for handling different types of FPDEs [20–29]. Moreover, it is an efficient method to find out the coefficients of the series solutions. Construction of multidimensional and multiple solutions for fractional differential equations in the form of power series is an important advantage of RPSM [28]. RPSM is effective and easy to use for solving linear and nonlinear FPDEs without linearization, perturbation, or discretization.

In the present paper, the RPSM will be implemented to construct a new algorithm for determining analytical solutions to the following space-time FPDE

$$D_t^{\alpha} u = D_x^{\beta} u + f(x,t), \tag{1}$$

* Corresponding author.
E-mail address: aylin@kocaeli.edu.tr (M. Aylin Bayrak).

https://doi.org/10.1016/j.amc.2018.04.032
0096-3003/

Nomenclature

$\Gamma(x)$	gamma function
$J^{\alpha}f(x)$	Riemann–Liouville fractional integral
$D^{\alpha}f(x)$	Caputo fractional derivative
$D_t^{\alpha}f(x)$	Caputo time fractional derivative
$D_x^{\beta}f(x)$	Caputo space fractional derivative
$f(x, t)$	source function

$$u(x,0)=\varphi(x),\ x\epsilon R \tag{2}$$

$$u(0,t)=\mu_1(t),\ t\epsilon R \tag{3}$$

$$u_x(0,t)=\mu_2(t),\ t\epsilon R, \tag{4}$$

where $0\le m-1<\alpha\le m,\ 0\le n-1<\beta\le n,\ m,n\epsilon N.$

2. Preliminaries

We first give the main definitions and various features of the fractional calculus theory in this section. The Riemann–Liouville fractional integral operator of order $\alpha\,(\alpha\ge 0)$ is defined as

$$J^{\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}f(t)dt,\ \alpha>0,\ x>0 \tag{5}$$

$$J^0 f(x)=f(x). \tag{6}$$

The Caputo fractional derivative of order α is defined as

$$D^{\alpha}f(x)=J^{m-\alpha}D^m f(x)=\frac{1}{\Gamma(m-\alpha)}\int_0^x (x-t)^{m-\alpha-1}f^{(m)}(t)dt,\quad m-1<\alpha\le m,\ x>0 \tag{7}$$

where D^m is the classical differential operator of order m.

For the Caputo derivative we have

$$D^{\alpha}x^{\beta}=0,\ \beta<\alpha \tag{8}$$

$$D^{\alpha}x^{\beta}=\frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\alpha)}x^{\beta-\alpha},\ \beta\ge\alpha. \tag{9}$$

Let n be the smallest integer greater than α, the Caputo time fractional derivative operator of order α of $u(x, t)$ is defined as [18–21]

$$D_t^{\alpha}u(x,t)=\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}=\begin{cases}\frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}\frac{\partial^n u(x,\tau)}{\partial t^n}d\tau, & n-1<\alpha\le n\\ \frac{\partial^n u(x,t)}{\partial t^n}, & \alpha=n\epsilon N\end{cases} \tag{10}$$

and the space fractional derivative of order β of $u(x, t)$ is defined as

$$D_x^{\beta}u(x,t)=\frac{\partial^{\beta}u(x,t)}{\partial x^{\beta}}=\begin{cases}\frac{1}{\Gamma(n-\beta)}\int_0^x (x-\tau)^{n-\beta-1}\frac{\partial^n u(x,\tau)}{\partial x^n}d\tau, & n-1<\beta\le n\\ \frac{\partial^n u(x,t)}{\partial x^n}, & \beta=n\epsilon N\end{cases} \tag{11}$$

The power series expansions about $t=t_0$ and $x=x_0$

$$\sum_{k=0}^{\infty}\sum_{l=0}^{m-1} f_{kl}(x)(t-t_0)^{k\alpha+l},\ 0\le m-1<\alpha\le m,\ t\ge t_0 \tag{12}$$

and

$$\sum_{k=0}^{\infty}\sum_{l=0}^{n-1} g_{kl}(t)(x-x_0)^{k\alpha+l},\ 0\le n-1<\alpha\le n,\ x\ge x_0 \tag{13}$$

are called multiple fractional power series, where $f_{kl}(x)$ and $g_{kl}(t)$ are called the coefficients of the series.