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# Stability of Markovian jump stochastic parabolic Itô equations with generally uncertain transition rates

Caihong Zhang[a,*], Yonggui Kao[b,*], Binghua Kao[c], Tiezhu Zhang[d]

[a] College of Automation and Electrical Engineering of Qingdao University, Qingdao 266071, PR China
[b] Department of Mathematics, Harbin Institute of Technology (Weihai), Weihai 264209, PR China
[c] Mathematics Science College, Inner Mongolia Normal University, Hohhot, 010022, PR China
[d] Shandong University of Technology, Zibo, 255000, China

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ABSTRACT

In this paper, the stability problem for delayed Markovian jump stochastic parabolic Itô equations (DMJSPIEs) subject to generally uncertain transition rates (GUTRs) is investigated via Lyapunov-Krasovskii functional and linear matrix inequality (LMI) method. In the model discussed, we suppose that only part of the transition rates of the jumping process are known, namely, some factors have been already available, some elements have been simply known with lower and upper bounds, and the rest of elements may have no useful information. Lastly, the applicability and effectiveness of the obtained results are illustrated through an example.



## 1. Introduction

The stability of Markovian jump stochastic parabolic Itô systems is fundamental problem due to its importance in analysis and synthesis of these systems. There have been a lot of researchers studying stability and control problems of dynamic systems which are governed by stochastic partial differential equations (SPDEs) with uncertain constant or time-varying delays [1–4]. Wang et al. [1] probed exponential stability of impulsive stochastic fuzzy reaction-diffusion Cohen–Grossberg neural networks with mixed delays by the cone theory. Luo and Zhang [2] considered almost sure exponential stability of stochastic reaction-diffusion system via Lyapunov direct method. Kao and Wang [3] analyzed global stability for stochastic coupled reaction-diffusion systems on networks by using the graph theory and some stability theory, with a close relation to the topological property of the networks. Recently, stochastic partial differential equations with Markovian switching (SPDEswMS) have attracted extensive attention, due to their wide popularity in describing a great number of systems, for example, power systems, communication systems, economics systems, fault-tolerant systems and so on (see for example [5–8] and the references therein). Wang et al. [6] considered stochastic exponential stability of the reaction-diffusion recurrent neural networks with time delay and Markovian jumping parameters. Vidhya and Balasubramaniam [8] discussed stability of the delayed stochastic parabolic Itô equation with Markovian jump. Yan et al. [9,10] used mode-dependent parameter approach and discussed finite-time stability and stabilization of Itô stochastic systems with Markovian switching. Shen et al. [11,12] considered reliable mixed passive and $H_\infty$ filtering for semi-Markov jump systems with randomly occurring uncertainties and sensor failures. Wang et al. [13,14] used the mismatched membership function approach and a sojourn probability approach to discuss the fuzzy-model-based reliable control for switched systems with mode-dependent

* Corresponding authors.
*E-mail addresses:* rainbow823@163.com (C. Zhang), kaoyonggui@sina.com (Y. Kao).

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time-varying delays respectively. The above mentioned results are all based on a comprehensive understanding of the conversion rates. However, in fact, the existing stability analysis does not fully use the properties of transition rates (TRs), which leads to the conservatism of the obtained stability criterion.

In practice, however, it is impossible to estimate the exact TRs [15] and it is a necessity to exploit the tactic for SPDEswMS with uncertain TRs. The uncertain TRs are described by three types at present, namely, bounded uncertain TRs (BUTRs). In [15], the stochastic stability for continuous-time and discrete-time Markovian jump linear systems (MJLSs) with upper bounded TRs was considered. The stability and stabilization for the continuous-time MJSs with partly unknown TRs (PUTRs) were discussed in [16]. PUTRs for MJSs were also involved in [17–27]. The stochastic stability analysis and delay-dependent $H_\infty$ filtering for singular discrete-time Markovian jump systems with partially unknown transition probabilities were studied by [25] and [26], respectively. In the aforementioned models, the complete knowledge of each TR is just either completely known or exactly unknown, which may not be guaranteed in practical situations. Besides, another description for the uncertain TRs called generally uncertain TRs (GUTRs) was considered by [28]. Every TR of this model can be totally unknown or only its estimate is known, thus we can apply this model to more realistic situations. Both partly uncertain and bounded uncertain TR models are the exceptional circumstances of GUTR models. kao et al. discussed global exponential stability of delayed Markovian jump fuzzy cellular neural networks and stabilisation of singular Markovian jump systems with generally incomplete transition probability, respectively in [29,30]. Cheng et al. [31] considered an asynchronous operation approach to event-triggered control for fuzzy Markovian jump systems with general switching policies. For all we know, exponential stability of the delayed stochastic parabolic Itô equation with Markovian jumping parameters and GUTRs is still remain open and has not been investigated yet.

Therefore, this paper deals with the exponential stability problem of the system subject to GUTRs. By constructing the Lyapunov-Krasovskii functional, a new criterion with less conservativeness is provided based on LMIs and can be lightly computed by the Matlab toolbox. The applicability and effectiveness of the obtained results are illustrated through an example. The proposed method be employed to handle T-S fuzzy systems in the following papers.

## 2. Model description

We consider the following stochastic parabolic Itô equations:

$$\begin{cases} \mathrm{d}u(x,t,w) = [\mathrm{div}(K(x,t)\circ \nabla u(x,t,w)) + A(r(t))g(u(x,t,w)) - M(r(t))u(x,t,w) \\ +C(r(t))u(x,t-\tau,w)]\mathrm{d}t + [g(u(x,t,w)) + g(u(x,t-\tau,w))]\mathrm{d}B(t,w),\ (x,t,w)\in \mathbb{D}\times\mathbb{R}^+\times\Omega; \\ u(x,0,w) = \phi(x,w)\in(u_0,u_1),\ (x,t)\in\mathbb{D}\times\Omega; \\ \frac{\partial u(x,t,w)}{\partial n(k)} = 0,\ (x,t,w)\in\partial\mathbb{D}\times\mathbb{R}^+\times\Omega; \end{cases} \tag{1}$$

When the process of mode jumping $\{r(t),\ t>0\}$ takes values in a finite state space $\mathcal{S}=\{1,2,\dots,s\}$ with the following transition probabilities, it is a right-continuous Markov process on the probability space.

$$\mathbb{P}r\{r_{t+\Delta}=j|r_t=i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i\neq j, \\ 1+\pi_{ij}\Delta + o(\Delta), & i=j, \end{cases} \tag{2}$$

where $\Delta>0$, $\lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta}=0$, and $\pi_{ij}\geq 0$, $(i\neq j)$ is the transition rate from mode $i$ at time $t$ to mode $j$ at time $t+\Delta$, and there is $\pi_{ii}=-\sum\limits_{j=1,j\neq i}^{s}\pi_{ij}\leq 0$. However, not all the transition rates is available or estimated. The mode transition rate matrix (TRM) $\Pi\triangleq(\pi_{ij})_{s\times s}$ is assumed to be commonly uncertain. For example, the TRM of system (1) with $s$ operation modes may be

$$\Pi = \begin{bmatrix} \hat{\pi}_{11}+\Delta_{11} & ? & ? & \cdots & \hat{\pi}_{1s}+\Delta_{1s} \\ ? & ? & \hat{\pi}_{23}+\Delta_{23} & \cdots & \hat{\pi}_{2s}+\Delta_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & ? & ? & \cdots & ? \end{bmatrix} \tag{3}$$

where $\hat{\pi}_{ij}$ and $\Delta_{ij}\in[-\delta_{ij},\delta_{ij}]$, $\delta_{ij}\geq 0$ stand for the estimate value and estimate error of the uncertain transition rate $\pi_{ij}$, respectively, and $\hat{\pi}_{ij}$, $\delta_{ij}$ are given. "?" is the totally unknown transition rate, which represents its estimate value $\hat{\pi}_{ij}$ and estimate error bound are unknown. At first, we have some assumptions and definitions. For all $i\in\mathcal{S}$, the set $U^i$ means $U^i=U^i_k\bigcup U^i_{uk}$, with $U^i_k\triangleq\{j:\ \pi_{ij}$ is known for $j\in\mathcal{S}\}$, $U^i_{uk}\triangleq\{j:\ \pi_{ij}$ is unknown for $j\in\mathcal{S}\}$. Besides, if $U^i_k\neq\emptyset$, it is described as $U^i_k=\{k^i_1,\dots,k^i_{m_i}\}$, where $k^i_{m_i}\in\mathbb{N}$ is the $m_i$th bound-known element with the index $k^i_{m_i}$ in the $i$th row of $\Pi$.

We have the following assumptions according to the properties of transition rates:

**Assumption 1.** If $U^i_k=\mathcal{S}$, then $\hat{\pi}_{ij}-\delta_{ij}\geq 0$, $(\forall j\in\mathcal{S},\ j\neq i)$, $\hat{\pi}_{ii}=-\sum_{j=1,j\neq i}^{s}\hat{\pi}_{ij}\leq 0$, and $\delta_{ii}=\sum_{j=1,j\neq i}^{s}\delta_{ij}>0$.

**Assumption 2.** If $U^i_k\neq\mathcal{S}$ and $i\in U^i_k$, then $\hat{\pi}_{ij}-\delta_{ij}\geq 0$, $(\forall j\in U^i_k,\ j\neq i)$, $\hat{\pi}_{ii}+\delta_{ii}\leq 0$, and $\sum_{j\in U^i_k}\hat{\pi}_{ij}\leq 0$.

**Assumption 3.** If $U^i_k\neq\mathcal{S}$ and $i\notin U^i_k$, then $\hat{\pi}_{ij}-\delta_{ij}\geq 0$, $(\forall j\in U^i_k)$.