Applied Mathematics and Computation 337 (2018) 428–441

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Numerical method for solving uncertain spring vibration equation

Lifen Jia[a], Waichon Lio[a], Xiangfeng Yang[b,*]

[a] Department of Mathematical Sciences, Tsinghua University, Beijing 100084, China
[b] School of Information Technology and Management, University of International Business and Economics, Beijing 100029, China

ARTICLE INFO

Keywords:
Spring vibration equation
Uncertain differential equation
Uncertainty theory
Uncertain process

ABSTRACT

As a type of uncertain differential equations, uncertain spring vibration equation is driven by Liu process. This paper proposes a concept of α-path, and shows that the solution of an uncertain spring vibration equation can be expressed by a family of solutions of second-order ordinary differential equations. This paper also proves that the inverse uncertainty distribution of solution of uncertain spring vibration equation is just the α-path of uncertain spring vibration equation, and a numerical algorithm is designed. Moreover, a formula to calculate the expected value of solution of uncertain spring vibration equation is derived. Finally, several numerical examples are provided to illustrate the efficiency of the numerical method.

1. Introduction

According to normality, duality, subadditivity and product axioms, uncertainty theory as a mathematical system was founded by Liu [6] in 2007, and perfected by Liu [8] in 2009. As an important contribution to uncertainty theory, uncertain process was proposed by Liu [7] for modelling the evolution of uncertain phenomena. As a counterpart of Wiener process, Liu process was first proposed by Liu [8] for dealing with white noise. It is a stationary and independent increment process whose increments are normal uncertain variables. Besides, almost all sample paths of Liu process are Lipschitz continuous. Based on Liu process, Liu [8] proposed uncertain calculus to deal with differentiation and integration of uncertain processes.

Uncertain differential equation was proposed by Liu [7] in 2008 driven by Liu process. Besides, Chen and Liu [1] first proved existence and uniqueness theorem for an uncertain differential equation. Later, the concept of stability of uncertain differential equation was proposed by Liu [8], some stability theorems were verified by Yao et al. [22], and stability in inverse uncertainty distribution was proved by Yang et al. [18]. More importantly, Yao and Chen [21] verified that the solution of an uncertain differential equation can be represented by a spectrum of solutions of ordinary differential equations. Following that, some other numerical methods such as Euler method (Yao and Chen [21]), Runge–Kutta method (Yang and Shen [13]), Adams method (Yang and Ralescu [14]), Adams–Simpson method (Wang et al. [11]), Milne method (Gao [2]), and Hamming method (Zhang et al. [23]) were also designed for solving uncertain differential equations. By now, uncertain differential equation has been widely applied in many fields such as finance (Liu [10]), optimal control (Zhu [24]), differential game (Yang and Gao [12,15]), heat conduction (Yang and Yao [16], Yang and Ni [17], Yang [19]), string vibration (Gao [3]), and spring vibration (Jia and Dai [4], Jia and Yang [5]). Uncertain differential equation has achieved fruitful results in both theory

* Corresponding author.
E-mail addresses: jlf17@mails.tsinghua.edu.cn (L. Jia), wj-liao16@mails.tsinghua.edu.cn (W. Lio), yangxf@uibe.edu.cn (X. Yang).

https://doi.org/10.1016/j.amc.2018.05.045
0096-3003/

and practice. For further explorations on the development and applications of uncertain differential equation, the interested readers may consult Yao's book [20].

As a sort of uncertain differential equation, uncertain spring vibration equation was first derived by Jia and Dai [4]. Since it is difficult to obtain the analytic solution of uncertain spring vibration equation, this paper will design a numerical method to solve uncertain spring vibration equation. The rest of this paper is arranged as follows. Section 2 reviews some basic concepts and theorems in uncertainty theory. Section 3 introduces uncertain spring vibration equation. Section 4 proposes a concept of α-path. Section 5 proves an important theorem. An inverse uncertainty distribution of solution theorem is proved in Section 6. Section 7 derives a formula to calculate the expected value of solution. In Section 8, we give several examples to illustrate the proposed numerical method. And Section 9 gives some conclusions.

2. Preliminaries

In this section, we review some basic concepts in uncertainty theory including uncertain variable, uncertain process and uncertain calculus.

For modelling belief degrees, Liu [6] defined an uncertain measure by the following three axioms:

Axiom 1. (Normality Axiom) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.

Axiom 2. (Duality Axiom) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.

Axiom 3. (Subadditivity Axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots,$ we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

Besides, Liu [8] defined a product uncertain measure by the fourth axiom:

Axiom 4. (Product Axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \ldots$. The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\},$$

where Λ_k are arbitrarily chosen events form $\mathcal{L}_k$ for $k = 1, 2, \ldots,$ respectively.

Theorem 2.1. *(Liu [9]) The uncertain measure is a monotone increasing set function. That is, for any events Λ_1 and Λ_2 with $\Lambda_1 \subset \Lambda_2$, we have*

$$\mathcal{M}\{\Lambda_1\} \le \mathcal{M}\{\Lambda_2\}.$$

Definition 2.1. (Liu [6]) Let Γ be a nonempty set, and let $\mathcal{L}$ be a σ-algebra over Γ. Then the triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

Definition 2.2. (Liu [6]) An uncertain variable ξ is a function from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that for any Borel set B of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$

is an event.

Definition 2.3. (Liu [6]) The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \le x\}$$

for any real number x.

An uncertainty distribution $\Phi(x)$ is said to be regular if it is a continuous and strictly increasing function with respect to x at which $0 < \Phi(x) < 1$, and

$$\lim_{x \to -\infty} \Phi(x) = 0, \qquad \lim_{x \to +\infty} \Phi(x) = 1.$$

Definition 2.4. (Liu [8]) The uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n} (\xi_i \in B_i)\right\} = \bigwedge_{i=1}^{n} \mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \ldots, B_n$ of real numbers.

Theorem 2.2. *(Liu [9]) Let $x_1, x_2, \ldots, x_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then*

$$x = f(x_1, x_2, \ldots, x_n)$$