Applied Mathematics and Computation 336 (2018) 272–287

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

High precision solution for thermo-elastic equations using stable node-based smoothed finite element method

Yanchun Tong

College of Mathematics and Statistics, Zhoukou Normal University, Zhoukou 466001, China

ARTICLE INFO

Keywords:
Thermo-elastic
Nodal integration
Gradient smoothing
Stable node-based smoothed finite element method (SNS-FEM)

ABSTRACT

The stable node-based smoothed finite element method (SNS-FEM) is presented to analyze thermo-elastic equations with various boundary conditions. The node-based smoothing domains are constructed to implement numerical integrations and then, the smoothed Galerkin weak form is utilized to obtain the discretized system equations. In the formulation, both the smoothed strains and the strain variance items over the node-based smoothing domains are used to perform the integration. Several numerical examples are studied in detail to investigate the performance of the SNS-FEM by comparison with the original NS-FEM and traditional FEM. Results show that the present SNS-FEM can provide very high precision solution for thermo-elastic equations. In addition, the SNS-FEM is more efficient than original NS-FEM and the standard FEM.

1. Introduction

The analysis of thermo-elastic equations is very important in engineering field. Heat induced warps and cracks will probably degrade or even destroy the components and structures. Therefore, the prediction of temperature gradient field and its resulted thermal stress field is of great importance [1–6]. As experimental study on these problems is usually very expensive and time-consuming, numerical methods have been widely employed, such as the Finite element method (FEM) [7–9], the element-free Galerkin method (EFG) [10–12], the meshless local Petrov–Galerkin method (MLPG) [13–16], point interpolation method (PIM) [17–18], the boundary element method (BEM) [19–21], etc.

In the past several decades, the FEM has become one of the most widely used numerical methods with many commercial software packages available [22]. However, there is an inherent defect for the fully compatible FEM model, which is called the “overly-stiff” property. Due to this, significant errors can occur especially in high gradient regions. To tackle these problems, Liu [23] has applied the gradient smoothing technique to the standard FEM. Then, a set of smoothed finite element methods have been formulated, including the cell-based smoothed finite element method (SFEM or CS-FEM) [24–29], the edge-based smoothed finite element method (ES-FEM) [30–34], the face-based smoothed finite element method (FS-FEM) [35–38] and the node-based smoothed finite element method (NS-FEM) [39–42].

In the NS-FEM, Gradient smoothing technique and node-based smoothing domains are utilized to modify the gradient fields and to perform the integration required in the weak form formulation. The node-based gradient smoothing operation makes NS-FEM well cure the “overly-stiff” phenomenon of the standard FEM. It is found that the NS-FEM works well with triangular and tetrahedral meshes that can be generated automatically. The NS-FEM is also free from volumetric locking and

E-mail address: Tongchunyan81@163.com

https://doi.org/10.1016/j.amc.2018.05.006
0096-3003/

is less sensitive to mesh distortion. In addition, the NS-FEM is more efficient than traditional FEM and has the upper bound property in the strain energy when a reasonably fine mesh is used.

It should be noted here that the NS-FEM can be viewed as a special case of the node-based point interpolation method (NS-PIM). When using linear triangular and tetrahedral meshes, the NS-FEM and the NS-PIM are equivalent. The NS-PIM was originally formulated for elasticity problems [43], and was then developed by Wu et al. for steady heat transfer [44] and thermo-elastic problems [45,46]. It is found that the NS-PIM performs well in analysis of these problems, and has quite similar properties as the NS-FEM. However, it has been found that both NS-FEM and NS-PIM exhibit temporal instability dealing with dynamic problems due to their "under-integration" induced by the excessive node-based smoothing operation. As a result, the application of the NS-FEM (or NS-PIM) to the analysis of dynamic problems is limited.

To tackle this problem, Beissel and Belytschko [47] first proposed the squared-residual stabilization technique, which was later developed by Zhang and Liu [48] and Feng et al. [49]. Besides, some researchers formulated the alpha finite element method (αFEM) [50] and the hybrid smoothed finite element method (H-SFEM) [51], by combination of the standard FEM and the NS-FEM. In these works, a parameter α is introduced and it is found that these methods can provide "ultra-accurate" numerical solutions with a proper parameter α. However, researchers have not found an optimal α value suitable for all problems until now.

Recently, a new stable node-based smoothed finite element method (SNS-FEM) without introducing an uncertain parameter has been formulated by Cui et al., which can cure the instability of NS-FEM and further improve the accuracy. The SNS-FEM has been successfully applied to acoustic problems [52,53], solid mechanic [54], stochastic analysis [55], electromagnetic analysis [56–58] and transient heat transfer problems [59]. For heat transfer analyses using the SNS-FEM, both temperature gradients and variance of temperature gradients of each node-based smoothing domain are taken into the Galerkin weak form. It turns out that the SNS-FEM can well eliminate "under-integration" of the NS-FEM and can provide stable solutions. Therefore, SNS-FEM is more accurate and efficient for heat transfer problems, compared with the FEM and the original NS-FEM.

Based on the studies of Cui and his co-workers, it is clearly seen that the SNS-FEM can be viewed as an excellent alternative to the original NS-FEM for engineering analysis. However, the SNS-FEM has not been extended to solving thermo-elastic equations until now. Special attention must be paid on thermal stress of structures induced by high temperature gradient in practical engineering and hence, the SNS-FEM is developed and extended to analyze 3-D thermo-elastic equations in this work. For thermo-elastic analysis using the SNS-FEM, except for the smoothed strains in each node-based smoothing domain, the variances of strains are also taken into consideration. Linear tetrahedral elements are used to discrete the problem domain and then, the generalized smoothed Galerkin weak form is utilized to obtain the discretized system equations. Numerical examples with different boundary conditions including both thermal and mechanical loads are studied to examine the accuracy and efficiency of present SNS-FEM through comparing the results with those obtained by the original NS-FEM and the standard FEM using the same mesh.

2. Thermo-elastic analysis using SNS-FEM

The governing equations and boundary conditions of thermo-elastic problems over the domain Ω bounded by Γ can be written as follows:

$$\sigma_{ij,j} + b_i = 0 \qquad \text{Problem domain studied} \tag{1}$$

$$u_i = u_\Gamma \qquad \text{Displacement boundary} \tag{2}$$

$$\sigma_{ij,j} = t_\Gamma \qquad \text{Stress boundary} \tag{3}$$

where u_Γ and t_Γ are the prescribed displacement and force, respectively. The heat and mechanics are linked by the following strain, stress, and thermal expansion relation:

$$\sigma_{ij} = \delta_{ij}\lambda\varepsilon_{kk} + 2\mu\varepsilon_{ij} - \delta_{ij}(3\lambda + 2\mu)\alpha\Delta T \tag{4}$$

where λ and μ are Lamé's constants which can be derived from elastic constants (ν and E), ΔT is the change in temperature and α is the thermal expansion. And the temperature field can be obtained by the following equations:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + Q(x, y, z) = 0 \qquad \text{Problem domain studied} \tag{5}$$

$$-k_x \frac{\partial T}{\partial x} n_x - k_y \frac{\partial T}{\partial y} n_y - k_z \frac{\partial T}{\partial z} n_z = q \qquad \text{Neumann or Heat flux boundary} \tag{6}$$

$$-k_x \frac{\partial T}{\partial x} n_x - k_y \frac{\partial T}{\partial y} n_y - k_z \frac{\partial T}{\partial z} n_z = h(T - T_a) \qquad \text{Robin or Heat convective boundary} \tag{7}$$

where k_x, k_y and k_z are the thermal conductivities, $Q(x, y, z)$ is the internal heat source, q is the prescribed heat flux, h is the convective coefficient and T_a is the temperature of surrounding medium.