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Stabilization of evolutionary networked games with length-r information ☆

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ABSTRACT

This paper investigates the dynamics of evolutionary networked games with different length information via semi-tensor product (STP) method. First, a networked game with different length information is modeled in the form of probabilistic Boolean networks (PBNs) with time delays. Second, based on the utility function of each player, a necessary condition for the existence of a pure Nash equilibrium is obtained. Then a state feedback control is applied to stabilize the considered system. Finally, an example is presented to substantiate the effectiveness of the theoretical results.

1. Introduction

In recent years, the study of evolutionary network games has triggered a hot discussion [1] motivated by the development of network science. Evolutionary games consider the interaction relationships among the individuals of the network [2,3], which interest a lot of researchers. In evolutionary game theory, the evolution of strategy profile is updated under the strategy update rules. Nowadays, evolutionary game theory has wide range of applications in many areas [4,5].

Evolutionary Boolean game is a special evolutionary network games and mainly used in systems science, biology and physics [6]. For example, Prisoner's dilemma [7], Snowdrift game and Hawk-Dove game are well-known Boolean games. There are usually two players in a Boolean game with a strategy set and everyone updates its strategy according to the payoff matrix. What is more, sometimes the history strategies of players and strategy profile dynamics are factors that determine the strategy updating rules [8,9]. The classical game as, e.g., the Prisoner's dilemma [7,9] can be modeled as a Boolean game. In this paper, we consider an evolutionary networked game with n players and each one has its own strategy set. As a result, any pair has an edge in the network has a payoff matrix. For each player, the history strategies information may have different influence on the updating. Hence a probability contribution is added to the model.

The traditional networked games often update strategy based on the previous step. However, in order to reduce the accidental error, in many cases, the strategies in the previous several steps need to be considered in real world, even more, there is a different degree of reference for the first steps. For example, in the managers and travelers game model, the managers have to take several history information of the travelers into consideration so as to make a proper strategy to improve the circulation efficiency of the network. We call a game with length-r information if each player in a game considers the

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forward r steps strategies to decide the strategy of this step. In [10], by modeling the two person Boolean game with fixed update rule, it used a long length information for linear fitting to update the players' strategies.

It should be noted that the model mentioned above is mainly numerical simulations, which does not apply to rigorous mathematical analysis for the game with long length information. Besides, each player is an independent individual and they are likely to consider different length information. This paper uses *STP* method to investigate the evolutionary Boolean games with different length information. *STP* method was first proposed by Cheng and his colleagues [11]. In recent years, the *STP* has been shown to be a powerful tool in many areas [12], especially in the study of logical networks, including stabilization [13,14,21], observability [15], controllability [16,17], reachability [18,19], system decomposition [20,22], optimization [23]. The Boolean network can be controlled by designing different controllers, for instance, pinning control [24–26], sampled-data control [27], feedback control [28]. In Boolean control networks, output regulation [29] and synchronization problems [30,31] have been studied. Besides, there are also some positive results in singular Boolean networks [32], probabilistic Boolean networks [33–35], impulsive networks [36], stochastic Boolean networks [37], logical control networks [39] and mixed-valued networks [38]. Moreover, by modeling the finite evolutionary game dynamics as a rigorous logical network, some good results on game theory have been obtained. For example, *STP* method has been applied to networked evolutionary games [40], including the analysis of the dynamics and Nash equilibriums [41], finding the potential functions of the potential games [42], stabilization [43] and optimization [44] for strategy profile of the evolutionary networked games. And in [45] the Markov-type evolutionary games (MtEGs) are studied by using the Lyapunov-based technique.

In this paper, we consider the evolutionary Boolean game with different length information via the *STP* method. Firstly, we turn the evolutionary Boolean game with different length information to a probabilistic Boolean networks (*PBN*) [46]. Secondly, a state feedback controller is considered to make the whole initial states globally converge to a stable strategy profile.

The rest of this paper is organized as follows. Section 2 gives some necessary preliminaries. Section 3 analyzes the dynamics of the evolutionary networked games with different length information and presents our main results. An illustrative example is presented in section 4 to support our main results. Section 5 is a brief conclusion.

2. Preliminaries

First, some notations are given as follows:

- $\Delta_n := \{\delta_n^k : 1 \le k \le n\}$, where δ_n^k is the kth column of identity matrix I_n.
- $\mathrm{Col}_i(M)$ ($\mathrm{Row}_i(M)$) is the ith column (row) of matrix M; $\mathrm{Col}(M)$ ($\mathrm{Row}(M)$) is the set of columns (rows) of M.
- A $m \times n$ matrix A is called a logical matrix if its columns $\mathrm{Col}(A)$ are of the form δ_m^i, that is $\mathrm{Col}(A) \subset \Delta_m$.
- $r = (r_1, \dots, r_k)^T$ is called a k-dimensional probabilistic vector if $r_i \ge 0, i = 1, \dots, k$ and $\sum_{i=1}^k r_i = 1$.
- The set of k dimensional probabilistic vectors is denoted by Υ_k.
- A $m \times n$ matrix B is called a probabilistic matrix if its columns $\mathrm{Col}(B)$ are m dimensional probabilistic vectors, that is $\mathrm{Col}(B) \subset \Upsilon_m$.
- The set of $m \times n$ probabilistic matrices is denoted by $\Upsilon_{m \times n}$.
- $\mathbf{1}_k$ and $\mathbf{0}_k$ denote the column vector of length k with all entries equaling 1 and 0, respectively.

Definition 1 [11]**.** The STP of two matrices $A \in R_{m \times n}$ and $B \in R_{p \times q}$ is defined as

$$A \ltimes B = (A \otimes I_{\frac{t}{n}})(B \otimes I_{\frac{t}{p}}) \in M_{(mt/n) \times (qt/p)}, \tag{1}$$

where $t = lcm(n, p)$ is the least common multiple of n and p, and $\otimes$ is the Kronecker product.

When $n = p$, $A \ltimes B = AB$. Therefore the STP is a generalization of the conventional matrix product. Symbol "$\ltimes''$ can be omitted and the matrix product is assumed to be STP in the sequel.

Proposition 1 *[11]. Let $x \in R^t$ be a column vector. Then for a matrix M*

$$xM = (I_t \otimes M)x. \tag{2}$$

Definition 2 [11]**.** For $A \in M_{m \times r}$, $B \in M_{n \times r}$, the Khatri–Rao product of A and B is defined as

$$A * B = [Col_1(A)Col_1(B), \dots, Col_r(A)Col_r(B)]. \tag{3}$$

Lemma 1 *[11]. Assume $X \in \Upsilon_p$, $Y \in \Upsilon_q$, we define two dummy matrices: (1) $D_r^{[p,q]}$, named by front-maintaining operator (FMO) and (2) $D_f^{[p,q]}$, named by rear-maintaining operator (RMO), respectively as follows:*

$$D_r^{[p,q]} = I_p \otimes 1_q^T, \quad D_f^{[p,q]} = 1_p^T \otimes I_q, \tag{4}$$

then we have:

$$D_r^{[p,q]}XY = X, \quad D_f^{[p,q]}XY = Y. \tag{5}$$