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An efficient numerical algorithm for the fractional Drinfeld–Sokolov–Wilson equation

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ABSTRACT

The fundamental purpose of the present paper is to apply an effective numerical algorithm based on the mixture of homotopy analysis technique, Sumudu transform approach and homotopy polynomials to obtain the approximate solution of a nonlinear fractional Drinfeld–Sokolov–Wilson equation. The nonlinear Drinfeld–Sokolov–Wilson equation naturally occurs in dispersive water waves. The uniqueness and convergence analysis are shown for the suggested technique. The convergence of the solution is fixed and managed by auxiliary parameter $\hbar$. The numerical results are shown graphically. Results obtained by the application of the technique disclose that the suggested scheme is very accurate, flexible, effective and simple to use.

1. Introduction

Fractional calculus is employed in various fields such as physics, mathematical biology, viscoelasticity, physics, signal processing, electrochemistry, finance, social science and many more. In this special branch, derivatives and integrals of fractional order are important aspects. It has been shown by many researchers that fractional generalizations of integer order models describe the natural phenomena in a very efficient manner (see the monograms [1–5]). The classical derivatives are in local nature whereas the Caputo fractional derivative is in nonlocal nature i.e., employing the classical derivatives we can analyze changes in a neighborhood of a point but applying Caputo fractional derivatives we can analyze changes in an interval. Due to this property the Caputo fractional derivative suitable to simulate more physical phenomena such as ocean climate, atmospheric physics, dynamical systems, earthquake, vibrations, polymers etc. (see the research papers [6–11]).

The nonlinear partial differential equations (NPDE) of mathematical physics are key issues in physical sciences. In the analysis of nonlinear physical phenomena the study of solutions for nonlinear evolution problems of fractional order plays a major role. Hence, the authors are encouraged to investigate the fractional nonlinear Drinfeld–Sokolov–Wilson (DSW) equation associated with Caputo fractional derivative presented as

$$\begin{cases} D_\eta^\mu v + \gamma w w_\xi = 0,\ 0 < \mu \le 1 \\ D_\eta^\mu w + \lambda w_{\xi\xi\xi} + \tau v w_\xi + \sigma v_\xi w = 0, 0 < \mu \le 1, \end{cases} \tag{1}$$

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where $v = v(\xi, \eta)$, $w = w(\xi, \eta)$ are variables, γ, λ, τ and σ are nonzero parameters. $D_\eta^\mu v$ and $D_\eta^\mu w$ represent the fractional order derivatives of $v(\xi, \eta)$ and $w(\xi, \eta)$ in Caputo sense. If we take $\mu = 1$ then Eq. (1) reduces to the standard DSW system which was firstly introduced by Drinfeld and Sokolov [12,13] and Wilson [14] when $\gamma = 1$, $\lambda = \tau = 2$, $\sigma = 1$. The fractional nonlinear DSW system considered as a mathematical model of dispersive water waves and it plays very important role in fluid mechanics [12–14]. The different methods are used by several authors to solve DSW equation [15–18]. Usually, there exists no algorithm that provokes an exact solution for NPDE involving the Caputo and Riemann–Liouville fractional derivative. By applying linearization or successive or perturbation schemes only approximate solutions can be obtained. The NPDE was evaluated by exerting various techniques [19–21] with their own weakness and limitations such as huge computational work, accounting of much high time and divergent results. In 1992, Chinese mathematician Liao suggested an analytic scheme namely hamotopy analysis method (HAM) [22–24] for solving the NDE.

The standard analytic schemes require more computer memory and time for computational work. Hence, to deal with these restraints of analytical schemes there is a need of amalgamation of these techniques with another existing scheme. In the present discussion, we investigate the nonlinear fractional DSW equation by exerting homotopy analysis Sumusu transform technique (HASTM) [25–27]. The HASTM is a novel and efficient amalgamation of the HAM [22–24], homotopy polynomials and standard Sumudu transform technique [28]. It is very interesting to observe that the technique is a mixture of two powerful computational techniques for handling nonlinear fractional problems. The HASTM solution involves an auxiliary parameter $\hbar$, with the aid of this parameter we can manage the convergence of the solution. The development of the article is given as: In Section 2, preliminaries of derivatives and integrals of fractional order and Sumudu transform is presented. In Section 3, the basic plan of HASTM is discussed. In Section 4, a detailed analysis of the solution is studied. In Section 5, solution of fractional DSW equation is investigated. Numerical simulation and discussions are provided in Section 6. Section 7 points out the concluding remarks.

2. Preliminaries

In this portion, we explore the preliminaries pertaining to integrals and derivatives of arbitrary order and Sumudu transform.

Definition 1. Let a function $g(\eta) \in C_\alpha, \alpha \geq -1$, then the left-sided Riemann–Liouville fractional integral of order $\mu > 0$ is given as [1]:

$$J^\mu g(\eta) = \frac{1}{\Gamma(\mu)} \int_0^t (\eta - \varsigma)^{\mu-1} g(\varsigma) d\varsigma, \quad (\mu > 0), \tag{2}$$

$$J^0 g(\eta) = g(\eta). \tag{3}$$

The following result holds for the fractional integral operator J^μ

$$J^\mu \eta^\vartheta = \frac{\Gamma(\vartheta + 1)}{\Gamma(\vartheta + \mu + 1)} \eta^{\mu+\vartheta}. \tag{4}$$

Definition 2. The Caputo fractional operator of $g(\eta)$ is presented as [2]:

$$\begin{aligned} D_\eta^\mu g(\eta) &= J^{k-\mu} D^k g(\eta) \\ &= \frac{1}{\Gamma(k - \mu)} \int_0^\eta (\eta - \varsigma)^{k-\mu-1} g^{(k)}(\varsigma) d\varsigma, \end{aligned} \tag{5}$$

for $k - 1 < \mu \leq k, \quad k \in N, \quad \eta > 0.$

The following result is very useful

$$J_\eta^\mu D_\eta^\mu g(\eta) = g(\eta) - \sum_{s=0}^{k-1} g^{(s)}(0+) \frac{\eta^s}{s!}. \tag{6}$$

Definition 3. There are many integral transforms available in the literature like Laplace, Fourier, Hankel etc., in this sequence the Sumudu transform (ST) is a recently proposed integral transform. This transform was firstly discovered and nurtured by Watugala [28]. The ST is explained and described over the set of functions

$$\Theta = \{g(\eta) : \exists N, \delta_1, \delta_2 > 0, \ |g(\eta)| < N e^{|\eta|/\delta_j}, \ \text{if} \eta \in (-1)^j \times [0, \infty)\}$$

as follows

$$G(u) = S[g(\eta)] = \int_0^\infty g(u\eta) e^{-\eta} d\eta, u \in (-\delta_1, \delta_2). \tag{7}$$

One can see the detailed properties and applications of ST in number of papers [29–32]. The ST of $D_\eta^\mu g(\eta)$ is given as follows [33]

$$S[D_\eta^\mu g(\eta)] = u^{-\mu} S[g(\eta)] - \sum_{s=0}^{k-1} u^{-\mu+s} g^{(s)}(0+), \ (k - 1 < \mu \leq k). \tag{8}$$