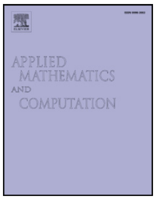
Applied Mathematics and Computation 335 (2018) 38–49

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc



Output synchronization control for a class of complex dynamical networks with non-identical dynamics

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ARTICLE INFO

Keywords:
Complex dynamical networks (CDNs)
Output synchronization
Algebraic Riccati equation (ARE)
Global Lyapunov function

ABSTRACT

This paper is concerned with the output synchronization control problem for a class of complex dynamical networks (CDNs) with non-identical dynamics. An output feedback control protocol consisting of a feedforward control law and a feedback control law is developed to achieve the output synchronization. More specifically, the feedforward control law is designed to compensate the coupling dynamics of the CDNs, and the feedback control law is designed by solving an algebraic Riccati equation (ARE), which is established by defining a modified quadratic performance index. It is shown that the output feedback control protocol solves the output synchronization control problem, and the graph theory is used to construct a global Lyapunov function, based on which a rigorous asymptotic convergence analysis of output synchronization errors is conducted. Finally, a simulation example is given to verify the effectiveness of the theoretical results.

1. Introduction

CDNs are developed in order that each node's dynamical behavior and information can be communicated in an effective way [1–9]. Over the past decades, since the CDN is a typical collective behavior in nature, it becomes an important study direction. Moreover, a rich body of literature concerned with CDNs has been proposed, motivated by CDNs' widespread applications in biology, physics, social sciences and engineering [10–16]. In particular, there has been an increasing interest in developing synchronization control strategies [17–26] in order that the experience can be shared by all nodes of a CDN system. For instance, global synchronization and asymptotic stability are investigated in [23], where a reference state is combined with the Lyapunov theorem. Several adaptive control criteria that ensure local and global synchronization of a CDN are derived in [24]. Moreover, the analysis of state synchronization control for a class of CDNs with actuator faults are both studied in [25,26]. Although the aforementioned synchronization control approaches have been proved effective, all of these methods focus on state synchronization.

A rich body of literature [27–32] has been devoted to the study of output synchronization control for complex networks. For example, a sufficient condition for output consensus of a CDN with fixed coupling strength was introduced in [30]. A consensus protocol proposed in [31], which is based on the relative output measurements between neighbors, was applied to

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https://doi.org/10.1016/j.amc.2018.04.029
0096-3003/

addressing output synchronization problem. Additionally, the output synchronization problem of a coupled neural networks with external disturbances was solved in the literature [32]. Note that the system dynamic matrices in [27–32] are all required to be identical. However, such a requirement is restrictive for some control systems. Therefore, it is desired to design control policies for complex networks with non-identical dynamics.

Recently, the synchronization control problem for multi-agent systems with non-identical dynamics has drawn much attention [33–39]. For example, in [35,36], adaptive distributed controllers were designed to cope with output tracking control problem. Nevertheless, adaptive approaches can only guarantee a bounded synchronization error. In order to achieve asymptotic synchronization, a distributed leader–follower algorithm was investigated in [37] where the internal model principle was used. In addition, output synchronization of heterogeneous systems under time-varying and directed interconnection structures was studied in [38], and a general framework was established in [39] for handling the output regulation. It is worth mentioning that how to compensate the coupling dynamics is not considered in [33–39]. In fact, for a CDN system the coupling dynamics is involved in each node's dynamics and needed to be compensated to achieve synchronization. On the other hand, another common feature of [33–39] is that the state variables are required to be available, which may be a strict limitation. To our best knowledge, the output synchronization control problem for a CDN with non-identical dynamics has not been solved in the previous researches, which motivates the current study.

This paper is concerned with the output synchronization control problem for a class of CDNs with non-identical dynamics. An output feedback control protocol which consists of a feedforward control law and a feedback control law is developed. The coupling dynamics is compensated by designing the feedforward control law. And the feedback control law is developed based on the solution to an ARE, which is established by defining a modified quadratic performance index. It is shown that the output feedback control protocol ensures that the output of each node synchronizes to a reference trajectory. In addition, based on the global Lyapunov analysis approach, a rigorous asymptotic convergence analysis of output synchronized errors is conducted. Finally, a simulation analysis is provided to verify the proposed method.

The components of this paper are organized as the following. In Section 2, a CDN system description and the control objectives of this study are presented. A state feedback control protocol is designed in Section 3. An output feedback control scheme which achieves output synchronization is developed in Section 4. Section 5 provides the results of a simulation, and the final conclusion is summarized in Section 6.

Notations: The following symbols will be used throughout the whole paper. Symbol $\|\cdot\|$ is defined as the Euclidean norm for a vector. $\lambda_{min}(\cdot)$ represents the minimum eigenvalue for a matrix. The notation $f(t)\to 0$ means that a function $f(t)$ approaches zero when the time variable t reaches to infinity. Moreover, A^T is the transpose for a matrix A, and $He(A) = A + A^T$ holds if A is square. In addition, the identity matrix with appropriate dimension is expressed as I_i.

2. Preliminaries and problem statement

In this section, some essential concepts on graph theory are introduced. Also, the problem of output synchronization for a CDN is presented.

2.1. Graph theory

Some concepts on graph theory are given as follows. A digraph $\mathcal{G} = (V, E)$ contains a set $V = 1, 2, \ldots, N$ of vertices and a set $E = 1, 2, \ldots, M$ of arcs (i, j) leading from initial initial vertex i to terminal vertex j. Each arc $(i, j) \in E$ is associated with a nonzero real-valued weight l_{ij}. A diagraph $\mathcal{G}$ is assumed to be strongly connected if there exists a directed path from one node to another. Specifically, if there is a connection between the ith node and the jth node, $l_{ij} > 0$; otherwise, $l_{ij} = 0$. Furthermore, the Laplacian matrix of a digraph is defined as

$$L = \begin{bmatrix} \sum\limits_{k\neq 1} l_{1k} & -l_{12} & \cdots & -l_{1N} \\ -l_{21} & \sum\limits_{k\neq 2} l_{2k} & \cdots & -l_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -l_{N1} & -l_{N2} & \cdots & \sum\limits_{k\neq N} l_{Nk} \end{bmatrix} \tag{1}$$

and denote the cofactor β_i for the ith diagonal element of L. In addition, more detailed theories on the digraph, which is omitted here for brevity, can be found in [42].

2.2. Model description

The reference node dynamics is assumed to be

$$\dot{z}_0 = Sz_0 \tag{2}$$

$$y_0 = Gz_0 \tag{3}$$