Applied Mathematics and Computation 335 (2018) 50–64

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Motion of the vitreous humour in a deforming eye–fluid-structure interaction between a nonlinear elastic solid and viscoleastic fluid

Karel Tůma[a], Judith Stein[b], Vít Průša[a,*], Elfriede Friedmann[b]

[a] *Faculty of Mathematics and Physics, Charles University, Sokolovská 83, Praha 8, Karlín CZ 186 75, Czechia*
[b] *Institute of Applied Mathematics, Heidelberg University, Im Neuenheimer Feld 205, Heidelberg, DE 69120, Germany*

ARTICLE INFO

MSC:
76A05
74F10
92C10
76Z05

Keywords:
Viscoelastic fluid
Burgers model
Nonlinear elasticity
Fluid structure interaction
Eye

ABSTRACT

We study the motion of vitreous humour in a deforming eyeball. From the mechanical and computational perspective this is a task to solve a fluid-structure interaction problem between a complex viscoelastic fluid (vitreous humour) and a nonlinear elastic solid (sclera and lens). We propose a numerical methodology capable of handling the fluid-structure interaction problem, and we demonstrate its applicability via solving the corresponding governing equations in a realistic geometrical setting and for realistic parameter values. It is shown that the choice of the rheological model for the vitreous humour has a negligible influence on the overall flow pattern in the domain of interest, whilst it has a significant impact on the mechanical stress distribution in the domain of interest.

1. Introduction

The vitreous humour is a fluid-like material that occupies the space between the lens and the retina in the eyeball. It has several functions, some of them being of mechanical nature. In particular, the vitreous humour is essential in the transmission of the mechanical stresses in the eyeball, and it acts as a mechanical damper protecting the eye, see for example [1] and references therein. Consequently, the *mechanical properties* of the vitreous humour and its *motion* are important both in the understanding of the physiology and pathology of the eye.

The motion of the vitreous humour can be induced either by motion of the eyeball as the whole or by the deformation of the eyeball. In the latter case the vitreous humour interacts with the deforming sclera and the lens, which from the mechanical point of view constitutes a complicated fluid-structure interaction problem. The complexity of the problem rests in the fact that one needs to deal with an interaction between a non-Newtonian viscoelastic fluid (vitreous humour) and a nonlinear elastic solid (sclera and lens).

We present a numerical methodology that is capable of handling the fluid-structure interaction problem. Then we apply the methodology in numerically solving the governing equations in a setting that resembles the recent experiment by Shah et al. [2]. The numerical experiment shows that the adopted methodology can be used to predict key mechanical quantities such as the mechanical stress at the interface between the vitreous humour and the retina. These quantities are of interest

---

[*] Corresponding author.

E-mail addresses: ktuma@karlin.mff.cuni.cz (K. Tůma), judith.stein@iwr.uni-heidelberg.de (J. Stein), prusv@karlin.mff.cuni.cz (V. Průša), friedmann@iwr.uni-heidelberg.de (E. Friedmann).

https://doi.org/10.1016/j.amc.2018.04.030

in the study of several pathologies such as retinal detachment, which documents the usability of the proposed methodology in answering practically relevant questions.

The main contribution of the current study is twofold. First, the majority of the works focused on the motion of the vitreous humor have been so far focused on the saccadic eye movement (rapid oscillations of the eyeball as the whole), see the early study by David et al. [3] and numerous later studies by Repetto et al. [4–11] to name a few. In all these works, the geometry of the cavity filled by the vitreous humour is assumed to be *fixed*. In particular, the cavity is assumed to take either the spherical shape or a perturbed spherical shape. This is perfectly acceptable if the motion of the vitreous humour is *induced by the oscillation of the eyeball as the whole*, and important conclusion has been drawn in the referred works. However, if the motion of the vitreous humour is *induced by the deformation of the sclera*, then a completely different approach must be taken. The problem must be solved as a fluid-structure interaction between the deforming sclera/lens and the flowing vitreous humour.

Second, the early works focused on the motion of the vitreous humor predominantly assume that the vitreous humour is an incompressible Navier–Stokes fluid. This is a plausible assumption especially if the motion of pathological (liquefied) vitreous humour is of interest. The physiological vitreous humour is however known to exhibit viscoelastic properties, see the early study by Lee et al. [12] or a more recent study by Sharif-Kashani et al. [13]. Consequently, the impact of complex viscoelastic (non-Newtonian) rheology on the motion of the vitreous humour must be taken into account. Various relatively simple viscoelastic models have been recently studied in this respect. For example, [10] in their numerical study use the nonlinear viscoelastic model introduced (for polymeric fluids) by Giesekus [14]. On the other hand, yet more complex models have been introduced in order to fit the experimental data. In particular, [13] have described the mechanical response of the vitreous humour using a Burgers-type viscoelastic model, see [15]. So far, the vitreous humour motion predicted by this advanced model has been investigated neither by analytical nor by numerical methods.

In what follows we address both issues. First, we consider the flow of the vitreous humour in a *deforming cavity*. The deformation of the cavity is induced by the deformation of the sclera, while the deformation of the sclera is caused by an applied load. Second, the mechanical properties of the vitreous humour are in the present study described by a relatively complex *Burgers-type viscoleastic rate-type model*. Moreover, since we need to solve the problem for the deformation of the sclera, we also need a model for the response of this solid substance. Concerning the mechanical response of the sclera, we assume that it behaves as a (nonlinear) *hyperelastic solid*.

2. Outline

The through description of the problem geometry and boundary conditions is given in Section 3, whilst the problem setting resembles the recent experimental setting studied by Shah et al. [2]. The mathematical models used for the description of the response of the vitreous humour and sclera are introduced in detail in Section 4. Note that since the experimental data for the vitreous humour are interpreted using *one-dimensional* spring-dashpot analogues, see [13], we need to provide a three-dimensional variant of the one-dimensional model. This step is not a straightforward one, and in addressing this issue we follow the thermodynamics based approach introduced by Rajagopal and Srinivasa [16].

Using the constitutive relations for the vitreous humour and the sclera and the lens, we finally formulate, see Section 5, the full system of governing equations for numerical simulation of the fluid-structure interaction problem. The numerical methodology for the solution of the arising fluid-structure interaction problem is discussed in Section 6; the methodology is based on the arbitrary Lagrangian–Eulerian method, see for example [17].

Finally, the proposed numerical method is used to solve the governing equations. In particular, see Section 7, we focus on comparison of the flow fields and the mechanical stress distributions obtained in two scenarios. In the first scenario we assume that the mechanical response of the vitreous humour is described by the standard Navier–Stokes model, while in the second scenario the vitreous humour is described by the Burgers-type viscoelastic model. It is shown that the choice of the rheological model for the vitreous humour has significant impact on the mechanical stress distribution in the domain of interest, whilst the overall flow field remains almost independent on the choice of the rheological model. In particular, the predicted mechanical stress at the interface between the vitreous humour and the retina vary significantly depending on the scenario used.

3. Problem description

In the experiments from Shah et al. [2] fresh bovine eyes were cut in an anterior-posterior direction to create approximately 2 cm thick samples with an optically clear window to analyze the changes during the experiment, see Fig. 1. Placed in an anterior-posterior orientation to the load cells the samples were attached at the sclera and near the lens by cotton swabs fixed via clamps to the load cells, Fig. 1. Then the samples were uniaxially stretched by simultaneously moving both load cells in 3 mm increments (up to 12 mm) with 120 s of equilibration time between each loading step, see Fig. 2. Mechanical strain was measured from sparse marker arrays on the surface of the vitreous and temporal collagen behavior was estimated from creep compliance rheological tests.