Stagnation point flow past a stretching/shrinking sheet driven by Arrhenius kinetics

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A R T I C L E   I N F O

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A B S T R A C T

The effect that a stagnation-point flow on a stretching/shrinking surface can have on an exothermic surface reaction is considered. The velocity of the surface relative to the outer flow is measured by the parameter λ with there being a critical value of λ. The surface reaction is described by the dimensionless reaction parameters α, β and ϵ, representing the heat of reaction, the reaction rate constant and the activation energy. The problem to determine how the dimensionless surface temperature θ_0 and concentration ϕ_0 varied can be simplified enabling θ_0 and ϕ_0 to be readily calculated in terms of reaction parameters. A hysteresis bifurcation is seen to arise distinguishing between cases where there are two critical points, and hence three solution branches, and where there is only a unique solution, the calculation of which placed an upper bound on the activation energy parameter ϵ for multiple solutions. The effect of the moving surface is seen to have a direct result on the surface temperature θ_0 and concentration ϕ_0 as well as on the locations of the critical values and the hysteresis bifurcation.

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1. Introduction

Stagnation-point flows, which describe the fluid motion near the stagnation region at the front of a blunt-nosed body, exist on all solid bodies moving in a fluid. The stagnation region encounters the highest pressure, the highest heat transfer and the highest rates of mass deposition. Hiemenz [1] was the first to study two-dimensional stagnation flows using a similarity transform to reduce the Navier–Stokes equations to nonlinear ordinary differential equations. The axisymmetric case was later solved by Homann [2]. Alternatively, stagnation point flows over stretching/shrinking surfaces have a wide range of applications in engineering and several technical purposes, particularly in metallurgy and polymer industry. For instance, the gradual cooling of a continuous stretching metal or plastic strips can be mentioned which have multiple applications in material production. The final quality of the product strongly depends on the rate of heat transfer from the stretching/shrinking surface, see Fisher [3] or Yu et al. [4] for example. Because of its numerous applications, the problem of the heat transfer in boundary-layer flows over stretching/shrinking surfaces has attracted much research.

It seems that Crane [5] was the first to present a self-similar solution in the closed analytical form for steady two-dimensional incompressible boundary-layer flow caused by a stretching surface whose velocity varies linearly with the

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distance from a fixed point on the surface. Following Crane's original study, the thermal approach to this problem was investigated by Carragher and Crane [6]. They assumed that the temperature difference between the surface and the ambient fluid (inviscid fluid) was proportional to a power of the distance from the fixed point. Banks [7] described a class of similarity solutions depending upon a parameter representing the power-law variation of the wall velocity for the boundary-layer equations due to a stretching wall. The existence and uniqueness of these stretching flow solutions is discussed by Mcleod and Rajagopal [8]. Extensive references on stretching surfaces in a viscous fluid have been presented by Wang [9]. The shrinking case of a viscous fluid has been first investigated by Miklavčič and Wang [10] followed by Fang [11] and Fang et al. [12]. The shrinking sheet flow, as discussed by Goldstein [13], is essentially a backward boundary-layer flow which shows different physical phenomena from stretching sheet flows.

In a series of papers, Merkin and Chaudhary [14], Chaudhary and Merkin [15,16], Ikeda et al. [17], Chaudhary et al. [18], Puri [19], Gray and Merkin [20,21], Gray et al. [22], Sadiq and Merkin [23], and Gray and Wake [24] have derived models for free convection boundary layers which are driven purely by the heat supplied to the surrounding fluid by an exothermic surface reaction. Here the reaction on the surface is modeled by single first-order kinetics with an Arrhenius temperature dependence. One feature of these models is the existence and the identification of the critical parameter values, a problem general in combustion systems, see for example Gray and Scott [25] and Scott [26]. These critical values often distinguish between cases where a steady state may be achieved and where there is ignition leading to thermal runaway.

The free convection boundary-1ayer flow on a vertical surface resulting from an exothermic catalytic chemical reaction on that surface is discussed by Merkin and Chaudhary [14]. The resulting system of partial differential equations is governed by the two dimensionless chemical parameters ϵ and α, which are measures of the activation energy and heat of reaction, respectively, as well as the Prandtl and Schmidt numbers. A series solution is obtained valid near the leading edge of the plate and this is continued downstream by numerical solutions of the full equations. These numerical solutions indicate the criticality of the system by local rapid increases in the reaction rate when α and ϵ are small. Asymptotic solutions valid at large distances downstream are obtained and these are shown to be essentially different in character between the cases when $\alpha = 0$ and when $\alpha > 0$. A singularity is seen to develop at a finite distance downstream when both $\epsilon = 0$ and $\alpha = 0$.

In further paper Chaudhary et al. [18] discuss a model for the free convection boundary-layer flow near the forward stagnation point driven by heating from a surface on which there is a catalytic reaction. Buoyancy-driven reactive boundary-layer flows have been treated by Puri [19], who considered both vertical and horizontal flows though in a somewhat different model to that treated in [14]. In [19] the reaction is assumed to take place within the boundary layer and is driven by evaporation from a pool of fluid on the wall. Solutions of the resulting boundary-layer problems are given but are restricted to a region close to the leading edge. Here we consider the case of a purely catalytic surface reaction in a forced convection stagnation-point flow on a moving surface. We examine how the fluid temperature and reactant concentration are affected by both the velocity of the moving wall and the catalytic surface reaction.

2. Equations

We consider the steady two-dimensional stagnation point flow of a viscous and incompressible fluid over a stretching/shrinking surface placed in the plane $y = 0$, where x and y are the Cartesian coordinates measured along the surface and normal to it, respectively. We assume that the fluid temperature and mass concentration within the flow are driven by the Arrhenius kinetics on the surface. It is also assumed that the velocity of the stretching/shrinking sheet is $U_w(x)$, while that of the far field (inviscid flow) is $U_\infty(x)$. An exothermic surface reaction is taking place on the wall which can be represented by the single first-order, non-isothermal, one-step reaction governed by Arrhenius kinetics of the following form, see Chaudhary et al. [18]:

$$A \rightarrow B \quad \text{rate } k_0 \, a \, e^{-E/RT}, \tag{1}$$

where a is the concentration of reactant A, T is the temperature of the fluid, E is the activation energy and R is the gas constant. The flow is assumed to be steady, laminar and incompressible with no heat generation within the boundary layer. Under these assumptions and neglecting viscous dissipation, the governing boundary-layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \nu \frac{\partial^2 u}{\partial y^2}, \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2}, \tag{4}$$

$$u \frac{\partial a}{\partial x} + v \frac{\partial a}{\partial y} = D \frac{\partial^2 a}{\partial y^2}, \tag{5}$$