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Compression and decompression based on discrete weighted transform

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ABSTRACT

The purpose of this paper is to introduce and extend the concept of the weighted transform and its inverse to functions of two variables. We show that they can be applied to image compression and decompression. The quality of the reconstructed image by inverse weighted transform is also compared with some known methods such as the fuzzy transform method and the standard JPEG method.

1. Introduction

In many problems, we are dealing with functions which have complex computation. So, it is reasonable to approximate functions by some methods which convert them into an n-dimensional vector (For more details see [1–3]). Fuzzy transform has useful applications in different fields such as, solution of ordinary and partial differential equations, image compression and decompression [5–7] and the references therein. Some authors have used fuzzy transform of functions of two variables to show that the quality of the reconstructed image is better than that of some others [1,4].

Regarding to Jahedi et al. [2], let φ be a continuous function from an arbitrary bounded subset E of $\mathbb{R}$ into (0, 1] and $\mathcal{B} = \{B_1, \ldots, B_n\}$ be a φ-partition of E. In addition, suppose that μ is a positive finite measure such that $\int_E B_k(x)\, d\mu(x) > 0$, for all $k = 1, \ldots, n$. The linear map

$$\mathcal{F}_{\mathcal{B}} : L^1(E, \mu) \to \mathbb{R}^n$$
$$\mathcal{F}_{\mathcal{B}}(f) = (F_1, \ldots, F_n)$$

is called the *F-transform based on a φ-partition* and is denoted by F_φ-transform of f. Here,

$$F_k = \frac{\int_E f(x) B_k(x) d\mu(x)}{\int_E B_k(x) d\mu(x)}, k = 1, \ldots, n.$$

To reconstruct the original function f, if $\mathcal{F}_{\mathcal{B}}(f) = (F_1, \ldots, F_n)$ is the F_φ-transform of f, then the linear operator $T_{\mathcal{B}} : L^1(E, \mu) \to C(E)$ defined by

$$T_{\mathcal{B}}(f) = \sum_{k=1}^{n} F_k \frac{B_k}{\varphi},$$

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is called the *inverse F_φ-transform of f with respect to $\mathcal{B}$*, where $C(E)$ is the space of real-valued of continuous functions on E. It is proved that the original continuous function can be approximated by the inverse F_φ-transform of f with respect to $\mathcal{B}$ with an arbitrary precision.

By introducing fuzzy transform of functions with two variables, some authors have realized that it can be applied in image compression so that the quality of reconstructed image will be better than the image compression based on fuzzy relation equation (ICF) method. It is also compared with the reconstructed image by using the standard JPEG method. In order to improve the quality of the reconstructed image by fuzzy transform, some authors proposed several algorithms such as FTR [8], LFTR [9], DFTR [10] and DSFTR [11].

In this paper, we introduce the concept of discrete weighted transform of two variables function, which is a little modification of weighted transform of one variable functions [3,12]. In Section 2, we prove that the original function can be approximated by the discrete inverse weighted transform with desired precision. Section 3 is devoted to the applications of discrete weighted transform in gray image compression and decompression. By applying the discrete weighted transform in the proposed algorithms, i.e. FTR, LFTR, DFTR, DSFTR, we show that the discrete weighted transform of two variables function is a powerful tool for gray image compression. In fact the quality of reconstructed image by inverse weighted transform is so convenient like some known methods such as the fuzzy transform and the standard JPEG methods.

2. Basic concepts and main results

Suppose that φ and ψ are continuous functions defined on $E = [a, b]$ and $F = [c, e]$ into $(0, d]$, $d \geq 1$, respectively.

Definition 1. Let $x_0 = x_1 < \cdots < x_n = x_{n+1}$ be nodes from E such that $x_1 = a$, $x_n = b$ and $n \geq 2$. The collection $\mathcal{B} = \{B_1, \ldots, B_n\}$ is called a φ-partition of E if each B_k is a continuous map from E into $[0, d]$, where $d \geq 1$, for all $k = 1, \ldots, n$ and satisfies in the following statements:

(i) $B_k(x_k) = \varphi(x_k)$, for all $k = 1, \ldots, n$.
(ii) $B_k(x) = 0$ whenever $x \notin (x_{k-1}, x_{k+1})$, for all $k = 1, \ldots, n$.
(iii) $\sum_{k=1}^{n} B_k(x) = \varphi(x)$ for all $x \in E$.

A φ-partition of E can be constructed by defining B_k, $k = 1, \ldots, n$, as follows:

$$B_1(x) = \begin{cases} \varphi(x)\left(\frac{x-x_2}{x_1-x_2}\right) & x \in [x_1, x_2] \\ 0 & otherwise; \end{cases}$$

$$B_k(x) = \begin{cases} \varphi(x) - B_{k-1}(x) & x \in [x_{k-1}, x_k] \\ \varphi(x)\left(\frac{x-x_{k+1}}{x_k-x_{k+1}}\right) & x \in [x_k, x_{k+1}] \\ 0 & otherwise; \end{cases}$$

for $k = 2, \ldots, n-1$ and

$$B_n(x) = \begin{cases} \varphi(x) - B_{n-1}(x) & x \in [x_{n-1}, x_n] \\ 0 & otherwise. \end{cases}$$

In the sequel, suppose that the original function f is known only at some nodes $(s_i, t_j) \in E \times F$, $i = 1, \ldots, N$ and $j = 1, \ldots, M$. Additionally, suppose that $S = \{s_1, \ldots, s_N\}$ and $T = \{t_1, \ldots, t_M\}$ are sufficiently dense with respect to the chosen partitions, i.e.:

$$(\forall k)(\exists i) B_k(s_i) > 0,$$
$$(\forall l)(\exists j) C_l(t_j) > 0$$

where $k = 1, \ldots, n$ and $l = 1, \ldots, m$.

Definition 2. Let f be a function given at nodes $(s_i, t_j) \in E \times F$, $i = 1, \ldots, N$, $j = 1, \ldots, M$. Let $\mathcal{B} = \{B_1, \ldots, B_n\}$ be a φ-partition of E and $\mathcal{C} = \{C_1, \ldots, C_m\}$ be a ψ-partition of F, where $n < N$, $m < M$. Suppose that the sets $S = \{s_1, \ldots, s_N\}$ and $T = \{t_1, \ldots, t_M\}$ are sufficiently dense with respect to the chosen partitions. Then the $n \times m$-matrix of real numbers defined as

$$\mathcal{F}_{nm}[f] = \begin{pmatrix} F_{11} & F_{12} & \ldots & F_{1m} \\ F_{21} & F_{22} & \ldots & F_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ F_{n1} & F_{n2} & \ldots & F_{nm} \end{pmatrix}_{n \times m} = \left(F_{kl}\right)_{l=1,\ldots,m}^{k=1,\ldots,n}$$

is called the discrete weighted transform of f with respect to $\mathcal{B} = \{B_1, \ldots, B_n\}$ and $\mathcal{C} = \{C_1, \ldots, C_m\}$, where

$$F_{kl} = \frac{\sum_{j=1}^{M} \sum_{i=1}^{N} f(s_i, t_j) B_k(s_i) C_l(t_j)}{\sum_{j=1}^{M} \sum_{i=1}^{N} B_k(s_i) C_l(t_j)}.$$