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Implicit-explicit one-leg methods for nonlinear stiff neutral equations

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A B S T R A C T

In this paper, by adapting the underlying implicit-explicit (IMEX) one-leg methods (cf. [1, 2]), a class of extended IMEX one-leg (EIEOL) methods are suggested for solving nonlinear stiff neutral equations (SNEs). It is proven under some suitable conditions that EIEOL methods are D-convergent of order 2 and stable for nonlinear SNEs. Several numerical examples are given to testify the obtained theoretical results and the computational effectiveness of EIEOL methods. Moreover, a comparison with the fully implicit one-leg methods is presented, which shows that EIEOL methods have the higher computational efficiency.

1. Introduction

In the last few decades, neutral equations have attracted considerable attention due to the wide range of applications in science and engineering, such as mechanics, bioscience, control theory and circuit analysis (see e.g. [3–5]). However, most of such equations cannot be solved analytically. Therefore, it is of great significance to construct the efficient numerical methods for solving this type of equations. Up to now, a lot of numerical methods for neutral equations have been derived. For example, collocation methods [6–8], discrete Runge–Kutta methods (cf. [9–12]), continuous Runge–Kutta methods (cf. [13–18]), linear multistep methods (cf. [19–21]), one-leg methods (cf. [22–26]) and so forth.

Some of the above research have dealt with the computation and analysis for stiff neutral equations (SNEs). However, most of the presentation devoted to the fully implicit methods, which lead to a large computational cost in general. To improve the computational efficiency of numerical methods, a good candidate is using the implicit-explicit (IMEX) splitting technique. For non-neutral equations, IMEX methods have been verified to be very effective and thus many interesting algorithmic results have been presented. For this topic, Ascher et al. [27] and Wang and Ruuth [28] constructed IMEX linear multistep methods and applied the methods to deal with time-dependent partial differential equations. Subsequently, Frank et al. [29] studied linear stability of IMEX linear multistep methods for ordinary differential equations (ODEs), Akrivis [30] and Li et al. [31] gave the adapted methods to nonlinear parabolic equations, Gjesdal [32] derived strong-stability-preserving algorithms, Hundsdorfer and Ruuth [33] involved hyperbolic equations with stiff sources or relaxation terms, in't Hout [34] discussed the numerical contractivity, Koto [35] extended the methods to solve delay differential equations (DDEs).

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Moreover, Xiao et al. [1] and Zhang and Xiao [2] investigated IMEX one-leg (IEOL) methods for stiff ODEs and non-neutral DDEs, respectively.

Although IMEX methods have been applied to many equations, to our knowledge, there have been no results dealing with nonlinear SNEs. Hence, motivated by the above research, in the presented paper we consider a class of extended IMEX one-leg (EIEOL) methods for solving the following complex or real d-dimensional initial value problems (IVPs) of SNEs:

$$\begin{cases} y'(t) = f(t, y(t)) + g(t, y(t), y(t-\tau), y'(t-\tau)), & t \in [t_0, T], \\ y(t) = \varphi(t), \quad t \in [t_0 - \tau, t_0], \end{cases} \tag{1.1}$$

where $y(t)$ is the unknown function, $\varphi(t) : [t_0 - \tau, t_0] \to \mathbb{C}^d$ is a continuously differentiable initial function, and $f : [t_0, T] \times \mathbb{C}^d \to \mathbb{C}^d$ and $g : [t_0, T] \times \mathbb{C}^d \times \mathbb{C}^d \times \mathbb{C}^d \to \mathbb{C}^d$ are two given sufficiently smooth mappings subject to following conditions for all $t \in [t_0, T]$ and $y, y_1, y_2, u, u_1, u_2, v, v_1, v_2, w \in \mathbb{C}^d$:

$$\Re\langle y_1 - y_2,\ f(t, y_1) - f(t, y_2)\rangle \le \mu_0 \|y_1 - y_2\|^2, \tag{1.2}$$

$$\|g(t, y_1, u_1, v_1) - g(t, y_2, u_2, v_2)\| \le \mu_1 \|y_1 - y_2\| + \mu_2 \|u_1 - u_2\| + \mu_3 \|v_1 - v_2\|, \tag{1.3}$$

$$\|H(t, y, u_1, v, w) - H(t, y, u_2, v, w)\| \le \mu_4 \|u_1 - u_2\|, \tag{1.4}$$

where $\langle \cdot, \cdot \rangle$ denotes an inner product on $\mathbb{C}^d$, $\|\cdot\|$ is the norm induced by this inner product, μ_0 and $\mu_i \ge 0$ $(i = 1, 2, 3, 4)$ are some given constants and

$$H(t, y, u, v, w) = g(t, y, u, f(t-\tau, u) + g(t-\tau, u, v, w)).$$

The paper is organized as follows. In Section 2, by adapting the underlying IEOL methods, a class of EIEOL methods for solving nonlinear SNEs are constructed. In Section 3, the global error of EIEOL methods is analyzed and thus it is proved under some suitable conditions that the EIEOL method is D-convergent of order 2. In Section 4, a numerical stability criterion is derived. Finally, in Section 5, several numerical examples are given to illustrate the obtained theoretical results and the computational effectiveness of the methods. The numerical results also show that the EIEOL methods are comparable.

2. The EIEOL methods

In this section, based on the underlying IEOL methods for ODEs, we will construct a class of EIEOL methods to solve nonlinear SNEs.

For the d-dimensional IVPs of ODEs with stiff term $f(t, y(t))$ and non-stiff term $g(t, y(t))$:

$$\begin{cases} y'(t) = f(t, y(t)) + g(t, y(t)), & t \in [t_0, T], \\ y(t_0) = y_0, \end{cases} \tag{2.1}$$

Xiao et al. [1] introduced the following IEOL methods:

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = hf\left(\sum_{j=0}^{k} \beta_j t_{n+j}, \sum_{j=0}^{k} \beta_j y_{n+j}\right) + hg\left(\sum_{j=0}^{k-1} \gamma_j t_{n+j}, \sum_{j=0}^{k-1} \gamma_j y_{n+j}\right), \qquad n \ge 0, \tag{2.2}$$

where $h > 0$ is the stepsize, $t_n = t_0 + nh, y_n \approx y(t_n)$, and α_j, β_j and γ_j are some real coefficients with $\alpha_k \beta_k \neq 0$. When introducing shift operator E: $Ey_n = y_{n+1}$ and polynomials

$$\rho(\xi) = \sum_{j=0}^{k} \alpha_j \xi^j, \quad \sigma(\xi) = \sum_{j=0}^{k} \beta_j \xi^j, \quad \varrho(\xi) = \sum_{j=0}^{k-1} \gamma_j \xi^j,$$

methods (2.2) can be written as

$$\rho(E) y_n = hf(\sigma(E) t_n, \sigma(E) y_n) + hg(\varrho(E) t_n, \varrho(E) y_n), \qquad n \ge 0, \tag{2.3}$$

where polynomials ρ, σ are assumed to have no common factor and satisfy consistent condition: $\rho(1) = 0$ and $\rho'(1) = \sigma(1) = \varrho(1) = 1$. According to references [1,2], an IEOL method is p-order consistent if the following conditions hold:

$$\begin{cases} \sum\limits_{j=0}^{k} \alpha_j = 0, \quad \sum\limits_{j=0}^{k} \frac{j^l}{l} \alpha_j = \left(\sum\limits_{j=0}^{k} j\beta_j\right)^{l-1} = \left(\sum\limits_{j=0}^{k-1} j\gamma_j\right)^{l-1}, \qquad l = 1, 2, \ldots, p, l \\ \left(\sum\limits_{j=0}^{k} j\beta_j\right)^{l} = \sum\limits_{j=0}^{k} j^l \beta_j, \quad \left(\sum\limits_{j=0}^{k-1} j\gamma_j\right)^{l} = \sum\limits_{j=0}^{k-1} j^l \gamma_j, \qquad l = 0, 1, \ldots, p-1. \quad l \end{cases} \tag{2.4}$$

By the above arguments, an IEOL method (ρ, σ, ϱ) can be viewed as the combination of an implicit one-leg method (ρ, σ) and an explicit one-leg method (ρ, ϱ).