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Regularity of uniform attractor for 3D non-autonomous Navier–Stokes–Voigt equation

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ABSTRACT

In this paper, we study the large time behavior for 3D viscoelastic incompressible fluid flow subject to Kelvin–Voigt damping and a time varying external force. The evolution of the dynamic is governed by a 3D non-autonomous Navier–Stokes–Voigt (NSV) equation. We assume that the external force is in the space of translation bounded functions (or its sub-spaces) that requires less compactness than the space of translation compact functions. We formulate the system in the framework of skew product flow. By defining an appropriate energy space that incorporates the Kelvin–Voigt damping, we established the existence of regular strong uniform attractor for the NSV equation in energy space $\widetilde{W}$ and provide a description of the general structure of the uniform attractor. Our result improves the H_0^1 regularity of global attractor of the system under study.

1. Introduction

Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with sufficiently smooth boundary $\partial\Omega$, we consider a 3D non-autonomous Navier–Stokes–Voigt (NSV for short, also Navier–Stokes–Voight) equation arising from the motion of viscoelastic incompressible fluid subject to Kelvin–Voigt damping, which was first introduced by Pavlovskii [33] (also Kelvin–Voigt model by Oskolkov [29])

$$\begin{cases} u_t - \nu\triangle u - \alpha^2\triangle u_t + (u\cdot\nabla)u + \nabla p = F(t,x) \text{ in } [\tau,+\infty)\times\Omega, \\ \nabla\cdot\ u = 0 \text{ in } [\tau,+\infty)\times\Omega, \\ u|_{\partial\Omega} = 0 \text{ on } [\tau,+\infty)\times\Omega,\ \ \tau\in\mathbb{R}, \\ u(x,\tau) = u_\tau(x),\ \ x\in\Omega, \end{cases} \tag{1.1}$$

here $u = u(t,x) = (u_1(t,x), u_2(t,x), u_3(t,x))$ is the velocity vector field, $p = p(t,x)$ is the scalar pressure, $\nu > 0$ is the coefficient of kinematic viscosity, the length scale α is a characterizing parameter of the elasticity of the fluid which describes the retardation time or the time of relaxation of deformations, $F(t, x)$ is the non-autonomous external force.

From the viewpoint of Kalantarov and Titi [20], (1.1) can be seen as a wave equation with damping, which has more regularity than classical Navier–Stokes equation. Cao et al. [7] proposed this system as a smooth, inviscid regularization of the 3D-Navier–Stokes equations.

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Based on the mathematical analysis of 2D and 3D Navier–Stokes equations in theory and computation by Ladyzenskaya [22], Oskolkov [29,30], Oskolkov and Shadiev [31] have proved existence of a unique almost classical solution in finite time interval for the system (1.1). Further investigations on solvability were proposed by Kalantarov [17,18], Cao et al. [7]. We refer to [1,2,5,11,21,23,28,32,44] and references therein for more results about regularity, decay, computation and applications of such models in organic polymer and food industry, and in the mechanisms of diffuse axonal injury. For recent theoretical development on more generalized Navier–Stokes–Voigt equation, such as with memory, we refer to works done by Gal and Medjo [13], Di Plinio et al. [34].

Let us recall some results of the long-time behavior of the Naver–Stokes–Voigt equations, the topic under study in this article. For the autonomous case, using semigroup decomposition method, it was shown in [20] that the continuous semigroup generated by the system (1.1) has a finite dimensional global attractor for $f \equiv 0$. In [19], Kalantarov et al. presented Gevrey regularity of attractors for the problem (1.1). In unbounded domain, Celebi et al. [8] proved the existence of global attractor of 2D case in weighted space. The existence of probability invariant measures associated with attractors of the 3D Navier–Stokes–Voigt equation has been established in Ramos and Titi [37]. Later et al. [24] used computation methods to show the statistical solutions of (1.1). For non-autonomous system, Yue and Zhong [41] using fractional power operator of Stokes operator A^{σ} and decomposition method to present uniform attractor in V by some asymptotic regularity estimates. Based on presentation with respect to universe of tempered sets, García et al. [15] derived compact or fixed family of pullback attractors. If the external force contains singularity, Qin et al. [35] proved the averaged singular limit between uniform and global attractors as singularity vanishes. Zelati and Gal [42] proved optimal regularity for global and exponential attractors which has finite-dimensionality described by 3D Grashof number. Moreover, the interesting result presented in [42] is the weak global attractor of 3D Navier–Stokes–Voigt equation subject to a non-time varying external force converging to 3D classical Navier–Stokes system as α tends to 0. For the stochastic dynamic systems of (1.1) in bounded domain and unbounded domain, such as random attractor and its Hausdorff dimension, we refer to [4,14,39]. Moreover, the exponential attractor and its finite dimensionality can be obtained by quasi-stable condition proposed by Chueshov and Lasiecka [10] and uniform stability of nonlinear fluid–structure interaction to a non-trivial equilibrium via Kelvin–Voigt type of damping is obtained in [26], in which the evolution of the interaction is governed by a system of PDEs coupling a Navier–Stokes equation with a wave equation

Although there are fruitful results about the attractors for (1.1) presented above and even Gevrey regularity in $(L^2(\Omega))^3$, as far as we know, the regularity of strong uniform attractors is still an open problem. Our objective in this paper is to investigate this problem. The main features and crucial technique can be summarized as following:

(i) A regular strong uniform attractor of (1.1) for 3D NSV equation is derived by energy inequality approach proposed by Ball [3].
(ii) By defining new energy and entropy space $\widetilde{V}$ and $\widetilde{W}$, respectively, that incorporate the Kelvin–Voight damping, we constructed some uniformly estimates for uniformly dissipation (absorbing) and uniformly asymptotic compactness, these spaces are more convenient to establish a priori estimate than the usual energy and entropy space.
(iii) For 3D autonomous NSV equation with periodic boundary condition, we show that the weak solution in $\widetilde{V}$ cannot become strong solution in $\widetilde{W}$ if we assume $u_0 \in V$ and $f \in H$ only, where the spaces H and V are defined as $H = \{u \in (L_2(\Omega))^3 | \text{div } u = 0\}$ and $V = H \cap (H_0^1(\Omega))^3$. This is a key difference between (1.1) and the 2D Navier–Stokes equation.
(iv) Moreover, the attractors of driving semigroup, semiflow, evolutionary process and non-autonomous dynamic systems are also established via the global attractor of skew product flow with some less regular external forces, which means the uniform attractors for non-autonomous dynamic systems and evolutionary processes coincide.

The plan of this paper is arranged as following. In Section 2, we state the well-posedness result of system (1.1) and make some remarks on periodic boundary condition. In Section 3, we review some important definitions and results from the theory of uniform attractors. In Section 4, we prove the main results of the article: the regularity of strong uniform attractor and its structure for skew product flow and comment on the existence of attractor for the driving semigroup, the skew-product semiflow, the evolutionary process generated from system (1.1) and the non-autonomous dynamic system by using theory reviewed in Section 3. At the end of Section 4, we provide a review on theory of attractors for non-autonomous dynamical system and discuss the possibility of using the similar method to establish results for system (1.1) in the framework of non-autonomous dynamical system.

2. Well-posedness of 3D non-autonomous NSV equation

2.1. Generalized functional spaces and some operators

Denote $E := \{u | u \in (C_0^{\infty}(\Omega))^3, \text{div } u = 0\}$, H defined above could be also viewed as the closure of the set E in $(L^2(\Omega))^3$ topology with inner product $(\cdot, \cdot)$ and norm $|\cdot|$

$$(u, v) = \sum_{j=1}^{3} \int_{\Omega} u_j(x) v_j(x) dx, \ \ |u|^2 = (u, u), \ \ \forall \ u, v \in H. \tag{2.1}$$