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A new iterative technique for a fractional model of nonlinear Zakharov–Kuznetsov equations via Sumudu transform

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ABSTRACT

The main objective of this paper is to suggest a new computational technique namely new iterative Sumudu transform method (NISTM) to solve numerically nonlinear time-fractional Zakharov–Kuznetsov (FZK) equation in two dimensions. We implemented the proposed technique on two test examples, plotted the solution and compared the absolute error with the variational iterative technique (VIM) and homotopy perturbation transform method (HPTM).

1. Introduction

Numerous important models are being modeled in various prominent areas like control [1,2], signal theory [3,4], mechanics [5,6], chemical [7,8], biological [9,10], acoustics [11,12], fluid [13–15] and in many other engineering and applied sciences using the fractional derivatives. We cannot imagine any model without fractional derivatives in daily life. There are many nonlinear models in this universe and precisely we can say that it is almost impossible to solve nonlinear fractional models analytically. So we have to find out solution numerically of the numerous nonlinear fractional models. There are many techniques which help us to find approximate solution as variational iterative method [16–21], homotopy perturbation method [22,23], homotopy analysis method [24], Laplace transform method [25–27], differential transform method [28] and q-homotopy analysis transform method [29] and many other methods.

This article studies new iterative Sumudu transform method to solve numerically nonlinear time-fractional Zakharov–Kuznetsov equation in two dimensions. The new iterative Sumudu transform method (NISTM) is a coupling of two methods of new iterative method (NIM) and Sumudu transform that gives solution in the form of convergent series in an easy way. In 2006, Daftardar-Gejji and Jafari [30] suggested a novel iterative technique which is applied by various authors to get the numerical solution of different classes of linear and nonlinear ordinary, partial and fractional order differential equations [31–35]. Jafari et al. [36] employed new iterative method to solve numerically different classes of fractional diffusion and fractional wave equations. It has also been applied by Bhalekar and Daftardar-Gejji to find numerical solution of fractional evolution equations and fractional boundary value problems [37].

In the present article, we study the ensuing nonlinear time-fractional Zakharov–Kuznetsov equations as given:

$$\frac{\partial^\beta u}{\partial t^\beta} + a\,\frac{\partial u^p}{\partial x} + b\,\frac{\partial^3 u^q}{\partial x^3} + c\,\frac{\partial^3 u^r}{\partial x \partial y^2} = 0,$$

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where u is a function of x, y and t, β is a parameter characterizing order of fractional derivative depending on time and $0 < \beta \leq 1$, where a, b, c are any real number constants and p, q and r belong to the set of natural numbers that manage the conduct of ion acoustic waves which are weakly nonlinear in a plasma consist of cold ions and hot isothermal electrons in the occurrence of a constant magnetic field [38]. This equation was obtained at first for expressing weakly ion-acoustic waves of nonlinear type in strongly magnetized lossless plasma in three dimensions [39]. It has been investigated in past years by many with the techniques like VIM [40], HPM [41] and HPTM [42] with their limitations. The key motivation of writing this article is that to put up a reliable computational technique to examine nonlinear fractional differential equations because of their utilities in mathematical modeling of real word problems in more accurate and systematic manner [43–45].

In this paper we propose the new iterative Sumudu transform method (NISTM) to solve numerically nonlinear FZK equations and before this paper new iterative method and Sumudu transform has been applied in many recent articles separately because of its strong properties [46–49]. We know very well that the integral transform method is very useful to find the solution of some type of linear and nonlinear ordinary and fractional partial differential equations. In the past literature there are many research work and articles on integral transforms such as Fourier, Mellin, Hankel, Laplace but very few research work on the power series transformation such as Sumudu transform, presumably so long as this transformation is meager known, and not extensively applied so far. Watugala [50–52] suggested a new integral transform namely Sumudu transform and its properties proven by [53], successively by Weerakoon [54,55] and applied by Eltayeb and Kılıcman [56] to find the solution of one dimensional wave equation with non-constant coefficients. In this article we have just combined these two methods and applied on two nonlinear problems. The Sumudu transform has many strong properties but most useful property is that it has some interesting advantages over the existing integral transforms, especially the 'unity' feature which could come up with convenience when we derive the solution of fractional or integer order differential equations. It is worth mentioning that semi-analytical techniques with Sumudu transform provide less C.P.U time to calculate nonlinear fractional models and problems which are used in many areas of applied science and numerous fields of engineering. The improvement of this method is its competency of joining dual strong techniques to solve numerically different classes of linear and nonlinear fractional order ordinary and partial differential equations. We can say that the projected approach can reduce the time and work of the computation in comparison to the established schemes while preserving great efficiency of the approximate results; the size decreasing amounts to an enhancement of the execution of technique.

2. Basic definition of Sumudu transform and fractional calculus

In present segment, we introduced fundamental terms and characterizations of Sumudu transform employed in fractional calculus to describe the proposed schemes.

Definition 1. The Sumudu transform can be defined on the domain of functions [46–49]:

$$B = \{g(p)|\exists N, p_1, p_2 > 0, |g(p)| < Ne^{\frac{|p|}{p_j}} \text{ if } p \in (-1)^j \times (0, \infty) \text{ as in the form of formulae}$$
$$S[g(p)] = \int_0^\infty g(up)e^{-p}dp, \ u \in (-p_1, p_2).$$

Definition 2. The Sumudu transform for Caputo fractional derivative can be defined as below [46–49]:

$$S[D_x^{m\alpha}u(x,t)] = s^{-m\alpha}S[u(x,t)] - \sum_{k=0}^{m-1} s^{(-m\alpha+k)}u^k(0,t), \ m-1 < m\alpha \leq m.$$

Definition 3. A real valued function $g(p)$, $p > 0$, is in the space $C_\alpha, \alpha \in R$, if there is a real number n, $(n > \alpha)$, such that $g(p) = p^n g_1(p)$, where $g_1(p) \in C[0, \infty)$ and it is in the space $C_\alpha^m \ if \ g^{(m)} \in C_\alpha, \ m \in N \cup \{0\}$ [57].

Definition 4. The fractional integral in the form of Riemann–Liouville of arbitrary order $\alpha \geq 0$ [57] of a given function $f(t) \in C_\beta, \beta \geq -1$ is defined as:

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}}\mathrm{d}\tau = \frac{1}{\Gamma(\alpha+1)}\int_0^t f(\tau)(\mathrm{d}\tau)^\alpha,$$
$$I^0 f(t) = f(t),$$

and Γ denotes Gamma function.

Definition 5. The Caputo fractional derivative of $f(t), f \in C_{-1}^m, m \in \mathrm{N}, \ m > 0$, [36–37] is defined as [50]

$$\mathrm{D}^\alpha f(t) = I^{m-\alpha}D^m f(t) = \frac{1}{\Gamma(m-\alpha)}\int_0^t (t-x)^{m-\alpha-1} f^m(x)dx,$$

where $m-1 < \alpha \leq m$.

The operator D^α has following basic properties:

(1) $D^\alpha I^\alpha f(t) = f(t)$,
(2) $I^\alpha D^\alpha f(t) = f(t) - \sum_{k=0}^{m-1} f^{(k)}(0^+)\frac{t}{\Gamma(k+1)}, m > 0.$