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Least-squares finite impulse response fixed-lag smoother and filter in linear discrete-time stochastic systems

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ABSTRACT

This paper proposes the least-squares (LS) finite impulse response (FIR) fixed-lag smoother and filter in linear discrete-time wide-sense stationary stochastic systems. The FIR fixed-lag smoothing estimate is given as a linear convolution sum of the impulse response function and the observed values. It is assumed that the signal is observed with additional white noise, which is uncorrelated with the signal process. By solving the simultaneous linear equations transformed from the Wiener–Hopf equation, the optimal impulse response function is obtained. The necessary information of the LS FIR fixed-lag smoothing algorithm is the auto-covariance function of the signal process and the variance of the observation noise process. In particular, this paper proposes the Levinson–Durbin algorithm, which needs less amount of arithmetic operations than the Gauss–Jordan elimination method in the inverse of the Toeplitz matrix, for the optimal impulse response function. From the numerical simulation example, the proposed LS FIR fixed-lag smoother and filter are superior in estimation accuracy to the RLS Wiener FIR estimators.

1. Introduction

There have been many studies on the signal and state estimation problems in linear and nonlinear stochastic systems. Recently, the resilient energy-to-peak filtering is considered for a class of uncertain continuous-time nonlinear systems [1]. The nonlinear system is described by the Takagi–Sugeno fuzzy model. The filtering error is asymptotically stable with guaranteed energy-to-peak filtering performance. In [2], the generalized H_2 filtering technique is proposed based on the linear matrix inequality in nonlinear discrete-time stochastic systems. In [3], the robust H_∞ filtering problem is studied in linear discrete-time stochastic systems with packet dropouts. In [4], for discrete-time Markov jump linear systems with multiplicative noises and measurement-delay, the finite-horizon estimator is proposed in the sense of minimum mean-square error. In [5], the finite-horizon filter is proposed for the multichannel and multiplicative noise systems with measurement delay.

This paper proposes the least-squares (LS) finite impulse response (FIR) fixed-lag smoother and filter, based on the Wiener–Hopf equation, in linear discrete-time stochastic systems. With regard to the FIR estimators, there are studies on the FIR filter [6,7], the receding horizon Kalman FIR filter [8–10], the FIR smoother [11] and the H_∞ FIR smoother [12], etc. In the discrete-time Kalman filter with the state-space models, the recursive least-squares (RLS) fixed-lag smoothing algorithm is devised by introducing the augmentation in the state equation [13]. In linear discrete-time stochastic systems, based on the RLS Wiener FIR filter [14], the RLS Wiener FIR fixed-lag smoothing and filtering algorithms [15] are proposed. Also, the RLS Wiener fixed-lag smoother is proposed, based on the invariant imbedding method, in linear discrete-time

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stochastic systems [16,17]. The RLS Wiener estimators use the information of the observation vector H, the system matrix Φ for the state vector $x(k)$ and the variance $K_x(k, k) = K_x(0)$ of the state vector. The system matrix is obtained by using the auto-covariance function of the signal process, which is approximated by the autoregressive (AR) model of order N.

Since the auto-covariance function $K(k-lag, s)$, $k-L \le s \le k$, of the signal process is included in the equation, transformed from the Wiener–Hopf equation, the derivation of the RLS Wiener FIR fixed-lag smoothing algorithm is not straightforward. For this reason, in [15], $K(k-lag, s)$, $1 \le lag \le N-1$, is expressed in terms of the linear combination of $K(k, s)$, $K(k+1, s)$, $K(k+2, s), \ldots, K(k+lag, s)$, $1 \le s \le k$, for the Nth order AR model. Here, lag denotes the fixed lag. By adopting this expression for $K(k-lag, s)$, the RLS FIR fixed-lag smoothing algorithm is derived based on the invariant imbedding method. However, as lag becomes large, the coefficients in the linear combination for $K(k-lag, s)$ become complicated expressions. Since the asymptotic stability conditions on the RLS Wiener FIR fixed-lag smoother and filter are not specified in [15], this paper shows the conditions. If the asymptotic stability conditions are not satisfied, as time advances, the fixed-lag smoothing and filtering estimates might no longer be optimal in linear least-squares estimation sense.

In this paper, the FIR fixed-lag smoothing estimate is given as a linear convolution sum of the impulse response function and the observed values. It is assumed that the signal is observed with the additional white observation noise, which is uncorrelated with the signal process. By solving the simultaneous linear equations, transformed from the Wiener–Hopf equation, the optimal impulse response function is obtained. The advantages of the proposed LS FIR fixed-lag smoothing and filtering algorithms are as follows.

(1) The proposed LS FIR fixed-lag smoother and filter require less information than the RLS Wiener estimators [15]. The auto-covariance function of the signal process, $K(i)$, $0 \le i \le L$, and the variance R of the observation noise are necessary for calculating the LS FIR fixed-lag smoothing and filtering algorithms.

(2) The LS FIR fixed-lag smoother and filter, based on the performance criterion (5), are optimal.

(3) The estimation accuracies of the proposed LS FIR fixed-lag smoother and filter are superior to those of the RLS Wiener FIR estimators [15], respectively, from the numerical simulation results.

(4) For the signal vector with m components, the simultaneous linear equations in matrix form involve $m \cdot (L+1)$ square Toeplitz matrix, which is assumed to be nonsingular. The optimal impulse response function for the fixed-lag smoothing estimate is calculated by applying the Gauss–Jordan elimination method to the matrix equation. For the $m \cdot (L+1)$ number of simultaneous linear equations, the Gauss–Jordan elimination method needs $O((m \cdot (L+1))^3)$ amount of arithmetic operations in the matrix inverse. From the property of the Toeplitz matrix, Theorem 3 proposes the Levinson–Durbin algorithm in calculating the optimal impulse response function for the fixed-lag smoothing estimate. The Levinson–Durbin recursion needs only $O(m \cdot (L+1))$ space. It is noteworthy that the Levinson–Durbin algorithm requires $O((m \cdot (L+1))^2) + O(m^3)$ amount of arithmetic operations [18] and is computationally faster than the Gauss–Jordan elimination method.

Section 4 presents the FIR fixed-lag smoothing error variance function to validate the stability of the proposed LS FIR fixed-lag smoothing algorithm. Section 5 demonstrates the numerical simulation example to show the estimation characteristics of the proposed LS FIR fixed-lag smoother and filter in comparison with the RLS Wiener FIR fixed-lag smoother and filter [15].

2. LS FIR fixed-lag smoothing problem

Let the observation equation and the state equation be described by

$$\begin{aligned} y(k) &= z(k) + v(k), \ z(k) = Hx(k), \\ x(k+1) &= \Phi x(k) + \Gamma w(k), \end{aligned} \tag{1}$$

in linear discrete-time wide-sense stationary stochastic systems. Here, $z(k)$ represents the m by 1 signal vector, $v(k)$ is the white observation noise, $x(k)$ is the n by 1 state vector, H is the m by n observation matrix, Φ is the system matrix or the state-transition matrix and $w(k)$ is the p by 1 white input noise. It is assumed that the observation noise process and the input noise process are mutually independent. Also, it is assumed that the signal process and the observation noise process are mutually independent and have zero means. Let the auto-covariance functions of $v(k)$ and $w(k)$ be given by

$$\begin{aligned} E[v(k)v^T(s)] &= R\delta_K(k-s), R > 0, \\ E[w(k)w^T(s)] &= Q\delta_K(k-s), Q > 0. \end{aligned} \tag{2}$$

Here, $\delta_K(k-s)$ denotes the Kronecker delta function. Let $K(k, s)$ represent the auto-covariance function of the signal process in the wide-sense stationary stochastic systems [19] and let $K(k, s)$ be expressed in the semi-degenerate kernel form of

$$K(k, s) = \begin{cases} A(k)B^T(s), 0 \le s \le k, \\ B(k)A^T(s), 0 \le k \le s. \end{cases} \tag{3}$$

Here, $A(k)$ and $B^T(s)$ are expressed as $A(k) = H\Phi^k$ and $B^T(s) = \Phi^{-s}K_x(s, s)H^T$. $K_x(s, s)$ represents the variance function of the state vector $x(k)$ and is equal to $K_x(0)$. Let the FIR fixed-lag smoothing estimate $\hat{z}(k-lag, k)$ of $z(k-lag)$ be given by

$$\hat{z}(k-lag, k) = \sum_{i=k-L}^{k} h(k, i)y(i) \tag{4}$$