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Optimal error estimate for energy-preserving splitting schemes for Maxwell's equations

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ABSTRACT

Two efficient splitting schemes are developed for 3D Maxwell's equations. The schemes are energy-preserving and unconditionally stable, while being implemented explicitly. Rigorous optimal error estimates are established for the proposed schemes, and especially the constant in the error estimates is only $\mathcal{O}(T)$. Numerical results confirm the theoretical analysis, and numerical comparison with some existing methods shows the good performance of the present schemes.

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1. Introduction

Maxwell's equations in an isotropic, homogeneous and lossless medium without charge and current densities are

$$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\epsilon}\mathbf{curl}\ \vec{H}, \quad \frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu}\mathbf{curl}\ \vec{E}, \quad \mathbf{div}(\epsilon \vec{E}) = 0, \quad \mathbf{div}(\mu \vec{H}) = 0, \tag{1.1}$$

where $\vec{E} = (E_x, E_y, E_z)^T$ and $\vec{H} = (H_x, H_y, H_z)^T$ are electric and magnetic fields, and constants ϵ and μ represent electric permittivity and magnetic permeability. The equations describe the relations of electric and magnetic, and have been widely used in engineering applications. In this paper, we consider the problem (1.1) with initial conditions $\vec{E}_0(x, y, z) = \vec{E}(x, y, z, 0)$ and $\vec{H}_0(x, y, z) = \vec{H}(x, y, z, 0)$ on the cuboid domain $\Omega = [a^-, a^+] \times [b^-, b^+] \times [c^-, c^+]$, and periodic boundary conditions on $\partial\Omega \times [0, T]$. Maxwell's Eq. (1.1) have energy conservation laws (ECLs)

$$\frac{d}{dt}\int_\Omega (\epsilon|\vec{E}|^2 + \mu|\vec{H}|^2)dxdydz = 0$$

and

$$\frac{d}{dt}\int_\Omega (\epsilon|\partial_t\vec{E}|^2 + \mu|\partial_t\vec{H}|^2)dxdydz = 0,$$

which are very important in a long-term propagation of electromagnetic waves. In addition, they also possess the symplectic [1,2] and multisymplectic [3,4] conservation laws. The schemes [3,5–8] have many attractive properties such as symplecticity, multisymplecticity, energy conservation and momentum conservations, while being inconvenient to use because of a large scale of computational costs and memory requirements. To improve the computational efficiency, the alternating direction implicit (ADI) method and local one-dimensional (LOD) technique are helpful. The ADI/LOD schemes

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[4,9–13] are efficient, but not energy-preserving. Recently, Chen et al. [14] developed energy-conserving and unconditionally stable schemes for 3D Maxwell's equations by combining the LOD technique with the spatial staggered grids. Other related works on the splitting method for Maxwell's equations can be referred to [15–17] and so on.

Most of the above-mentioned ADI/LOD schemes are of low-order accuracy in space, which means that we should refine the computational domain in order to obtain accurate solutions. For computing large scale problems, for problems requiring long-time integration, or for problems of wave propagations over longer distances, it is desirable to develop high-order schemes that produce smaller dispersion or phase errors for a given mesh resolution. The high-order compact (HOC) finite difference discretization [18] provides a better representation of shorter length scales. This feature brings it closer to the spectral/pseudospectral method, while the freedom in choosing the mesh geometry and the boundary conditions is maintained. Here we consider the HOC finite difference discretization in space. In present paper, we focus on the development of efficient energy-preserving splitting schemes for Maxwell's equations. An application of the LOD technique to Maxwell's equations yields splitting methods, in which each of the subproblems amounts to a series of mutually uncoupled 1D equations. Based on these obtained uncoupled 1D equations, we develop a series of splitting energy-preserving schemes for 3D Maxwell's equations. By the fast Fourier transform (FFT), the obtained schemes are represented as fully explicit forms. The intermediate variables in the splitting schemes bring difficulty to make a convergence analysis. To overcome the difficulty, the intermediate variables in the splitting schemes are reconstructed instead of being eliminated. Further making use of the energy method, we strictly establish the optimal error estimates for the proposed splitting schemes, in which the constant is only $\mathcal{O}(T)$. Some numerical experiments are conducted to show the performance of the proposed schemes.

The remainder of this paper is organized as follows. In Section 2, we introduce the time-splitting technique for Maxwell's equations. In Section 3, we briefly review the HOC finite difference discretization, and then apply the discrete FFT to them. Some splitting energy-preserving schemes are proposed in Section 4. Energy-preserving property and error estimate of the schemes are obtained in Sections 5 and 6, respectively. Finally, we make some conclusions in Section 7.

2. LOD technique for Maxwell's equations

Note that the **curl** operator can be split into the sum of two operators $\mathbf{curl}_+$ and $\mathbf{curl}_-$, i.e., $\mathbf{curl} = \mathbf{curl}_+ + \mathbf{curl}_-$, where

$$\mathbf{curl}_+ = \begin{pmatrix} 0 & 0 & \partial_y \\ \partial_z & 0 & 0 \\ 0 & \partial_x & 0 \end{pmatrix}, \quad \mathbf{curl}_- = \begin{pmatrix} 0 & -\partial_z & 0 \\ 0 & 0 & -\partial_x \\ -\partial_y & 0 & 0 \end{pmatrix},$$

and $\mathbf{curl}_\pm^* = \mathbf{curl}_\mp$. Let $\vec{u} = (\vec{E}^T, \vec{H}^T)^T$, Maxwell's Eq. (1.1) can be written in a shorter form

$$\partial_t \vec{u} = \boldsymbol{A}_- \vec{u} + \boldsymbol{A}_+ \vec{u}, \qquad \boldsymbol{A}_- = \begin{pmatrix} \mathbf{0} & \mathbf{curl}_- \\ -\mathbf{curl}_-^* & \mathbf{0} \end{pmatrix}, \quad \boldsymbol{A}_+ = \begin{pmatrix} \mathbf{0} & \mathbf{curl}_+ \\ -\mathbf{curl}_+^* & \mathbf{0} \end{pmatrix}. \tag{2.1}$$

The split-step approach leads us to repeatedly advance $\partial_t \vec{u} = \boldsymbol{A}_- \vec{u}$ and $\partial_t \vec{u} = \boldsymbol{A}_+ \vec{u}$ by certain time increments. If advancing the two subproblems by the time increments $\{\tau, \tau\}$, one obtains a splitting method which has first-order accuracy in time. One obtains second-order accuracy in time by instead alternating the two equations in the pattern $\boldsymbol{A}_-, \boldsymbol{A}_+, \boldsymbol{A}_-$ while using the time increments $\{\frac{\tau}{2}, \tau, \frac{\tau}{2}\}$– known as "Strang splitting". The two subproblems of the first-order method can be written out explicitly as:

- Stage 1.

$$\begin{cases} \varepsilon \frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x}, \\ \mu \frac{\partial H_z}{\partial t} = -\frac{\partial E_y}{\partial x}, \end{cases} \quad \begin{cases} \varepsilon \frac{\partial E_z}{\partial t} = -\frac{\partial H_x}{\partial y}, \\ \mu \frac{\partial H_x}{\partial t} = -\frac{\partial E_z}{\partial y}, \end{cases} \quad \begin{cases} \varepsilon \frac{\partial E_x}{\partial t} = -\frac{\partial H_y}{\partial z}, \\ \mu \frac{\partial H_y}{\partial t} = -\frac{\partial E_x}{\partial z}. \end{cases} \tag{2.2}$$

- Stage 2.

$$\begin{cases} \varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x}, \\ \mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}, \end{cases} \quad \begin{cases} \varepsilon \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y}, \\ \mu \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y}, \end{cases} \quad \begin{cases} \varepsilon \frac{\partial E_y}{\partial t} = \frac{\partial H_x}{\partial z}, \\ \mu \frac{\partial H_x}{\partial t} = \frac{\partial E_y}{\partial z}. \end{cases} \tag{2.3}$$

Note that each of the subproblems amounts to three pairs of mutually uncoupled 1D equations, and each of the 1D subsystems can easily be solved numerically.

3. Spatial discretization

Let $f(x, t)$ be an one-dimensional function defined on the interval $[a, b]$ with periodic boundary conditions. Consider a uniformly spaced mesh where the nodes are indexed by j. The independent variable at the nodes is $x_j = a + jh$, $0 \le j \le N-1$ and the function values at the nodes $f_j = f(x_j, t)$ are given. The HOC finite difference formula is [18]

$$\beta f'_{j-2} + \alpha f'_{j-1} + f'_j + \alpha f'_{j+1} + \beta f'_{j+2} = c\frac{f_{j+3} - f_{j-3}}{6h} + b\frac{f_{j+2} - f_{j-2}}{4h} + a\frac{f_{j+1} - f_{j-1}}{2h}, \tag{3.1}$$

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