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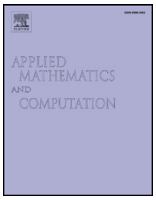

Induced-charge electro-osmosis in dielectric annuli

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ABSTRACT

This paper reports an analytical study on the induced-charge electro-osmosis (ICEO) within a leaky dielectric annulus subjected to an AC electric field. An interesting non-monotonic variation of the ICEO flow with the increasing AC frequency is revealed. This is different from the monotonic decrease of the ICEO flow around a cylinder submerged in an unbounded electrolyte solution upon increasing the AC frequency. Moreover, the ICEO flow is significantly reduced and may reverse direction due to the existence of the outer cylinder, depending on the charging responses of the annulus and the electrolyte solution, and the annulus geometry. In this analysis, we consider both the space charge layers (SCLs) and the electric double layers (EDLs) established within the solid and the liquid sides of the solid–liquid interfaces, respectively. The ICEO flow forms eight vortices within the annulus, which show a potential for mixing enhancement in micro/nanofluidics. As the AC phase increases, the ICEO flow changes periodically with a period half of the AC period. The outer cylinder presents a significant influence on the ICEO flow within the annulus since it affects the local electric fields and the induced zeta potentials of the cylinders. The present study may provide references for microchip fabrications with non-contact electrodes and biocell manipulations by electric fields.

1. Introduction

Electrokinetic phenomena have been widely studied because of their promising potential applications in various areas [1–3], among which the area of conventional linear electrokinetics has been extensively studied [4–6]. However, as the conventional linear electrokinetics only exists in DC electric fields due to its linearity to the applied electric fields, its practical application is limited. As a nonlinear electrokinetic phenomenon, induced-charge electrokinetics (ICEK) is receiving increasing attention recently due to its capability of net flow and particle motion generations in AC electric fields [7–9]. Various promising applications of ICEK have been proposed and studied, ranging from micromixers [10,11], micropumps [12,13], microvalves [14,15], micromotors [16], to the manipulations of particles [17,18] and droplets [19,20]. When a polarizable particle is submerged in an electrolyte solution under an electric field, it becomes polarized. The polarization surface charges attract counterions from the electrolyte solution and form an induced electric double layer (EDL) on the particle surface. The applied electric field exerts an electrical force on ions within the induced EDL and drives the ions into motion, which correspondingly leads to a bulk fluid flow referred to as induced-charge electro-osmosis (ICEO). The particle moves when the surrounding ICEO is asymmetric, referred to as induced-charge electrophoresis (ICEP). The zeta potential of the particle is linearly proportional to the applied electric field ($\zeta_i \propto E$) as it is induced by the applied electric field. Thus, the velocity

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of ICEO flow or ICEP particle motion is proportional to the applied electric field squared ($u \propto E^2$), which in turn ensures the existence of ICEK phenomena in AC electric fields.

ICEO flows have been experimentally captured on various conducting (ideally polarizable) surfaces, ranging from tin particles [21], copper structures [22], carbon structures [23], carbon steel spheres [24], gold-coated stainless steel cylinders [25–28], to pencil leads [29]. ICEP velocity of metallodielectric Janus particles has also been experimentally investigated [24,30,31]. Apart from these experimental investigations, numerous theoretical studies have also been carried out on ICEK phenomena on conducting (ideally polarizable) surfaces [32–36]. However, many materials in nature are dielectric and have finite polarizabilities. ICEK phenomena on dielectric (finitely polarizable) surfaces have not been widely studied yet. Lavrentovich et al. [37] and Ryzhkova et al. [38] experimentally studied the ICEP motions of dielectric particles in nematic liquid crystals. They found that the particles move in nematic liquid crystals even if they are ideally symmetric, and that the particle velocities are proportional to the third power of the electric field strength due to the anisotropic properties of the nematic liquid crystals. Zhao and Yang theoretically analyzed ICEO flows around a dielectric cylinder [39] and within a microchannel embedded with dielectric blocks [40]. They found that the ICEO flows reduce as the polarizability of the dielectric structures decreases. Boymelgreen and Miloh [41,42] theoretically studied the ICEP motions of dielectric Janus particles composed of two hemispheres of different permittivities, and found that the particle motions depend on the permittivities of each hemisphere and the ratio of them besides the EDL thickness. Hernández-Navarro et al. [43] experimentally studied the ICEP motions of microdroplets in nematic liquid crystals, and reported that the drug delivery could be realized through the controlled transport and coalesce of the microdroplets. Zhang and Li [44] numerically simulated the ICEP motions of dielectric particles in microchannels and found that the pair interaction of the particles is sensitive to the polarizabilities of the particles. In their later study on the ICEP motions of dielectric Janus particles composed of two hemispheres of different permittivities in microchannels, they found that Janus particles move in the microchannel with certain distance from the microchannel wall [45]. The distance depends on the polarizability ratio of the two hemispheres and the Janus particle size. They proposed to separate the Janus particles according to their polarizability ratios and sizes by leading them into different subchannels. Schnitzer and Yariv [46] developed a theoretical model of the ICEO flows on dielectric surfaces at moderate and large electric fields, and found that the dependence of ICEO flow on the electric field strength transits from a quadratic relationship to a linear relationship as the electric field strengths increase from moderate to strong.

The experiments of the aforementioned existing studies were conducted with the electrodes directly inserted into the electrolyte solutions, and the theoretical analyses simply assumed a background electric field surrounding the particles. However, as undesirable electrochemical reactions may occur on electrode surfaces, it is preferable to apply electric fields through the electrodes that are not in contact with the electrolyte solutions [47]. Thus, it is important to evaluate how an electric field enters a solid structure and generates ICEO flows within the structures. Therefore, in this paper, we analytically studied the ICEO flow around a leaky dielectric cylinder submerged in an electrolyte solution enclosed by a leaky dielectric cylindrical shell, i.e., the ICEO flow within a leaky dielectric annulus. A detailed analysis was carried out with wide ranges of various parameters. This study can provide guidelines to the microchip fabrications with non-contact electric fields. Moreover, it has been reported that electric fields induce electrical potentials on the biocell membranes [48,49]. Since the biocell membranes are polarizable, it is very likely that ICEO flows may be generated outside and inside the biocells, which may influence the biocell manipulations. Thus, the present study may also provide references to the biocell manipulations by electric fields.

2. Mathematical formulation

We consider a two-dimensional (2D) annulus composed of two leaky dielectric cylinders as shown in Fig. 1. The annulus is filled with a symmetric (z: z) electrolyte solution and subjected to a uniformly applied AC electric field $E = \mathrm{Re}\left(E_0 \exp(j\Omega t)\right)$, where z is the ion valence, Re( ) indicates the real part of the complex number, E_0 is the applied electric field strength, $j = \sqrt{-1}$ is the unit imaginary number, Ω is the dimensionless AC frequency scaled by the charging frequency of the electrolyte solution $t_f^{-1} = D_f/\lambda_{D,f}^2$. Here D_f and $\lambda_{D,f} = \sqrt{\varepsilon_0 \varepsilon_f k_B T/(2n_{0,f} z^2 e^2)}$ are the mass diffusivity of the ions and the EDL thickness, respectively, where ε_0 is the vacuum permittivity, ε_f is the dielectric constant of the electrolyte solution, k_B is the Boltzmann constant, T is the temperature of the electrolyte solution, $n_{0,f}$ is the bulk concentration of the ions in the electrolyte solution, and e is the elementary charge. Besides the EDLs, space charge layers (SCLs) are also developed in the solid sides of the solid–liquid interfaces [39]. The SCL thicknesses within the inner and the outer cylinders are $\lambda_{D,c} = \sqrt{\varepsilon_0 \varepsilon_c k_B T/(2n_{0,c} z^2 e^2)}$ and $\lambda_{D,s} = \sqrt{\varepsilon_0 \varepsilon_s k_B T/(2n_{0,s} z^2 e^2)}$, respectively, where ε_c and ε_s are the dielectric constants of the inner and the outer cylinders, respectively; and $n_{0,c}$ and $n_{0,s}$ are the bulk concentrations of the charge carriers within the inner and the outer cylinders, respectively.

The general solutions of the electrical potential within the inner cylinder ϕ_c, the electrolyte solution in the annulus ϕ_f, the outer cylinder ϕ_s, and outside the annulus ϕ_{out} are of the following forms:

$$\phi_c = -AEr\cos\theta, \tag{1}$$

$$\phi_f = -E\left(B_1 r + \frac{B_2}{r}\right)\cos\theta, \tag{2}$$