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Impact of network structure on synchronization of Hindmarsh–Rose neurons coupled in structured network

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ABSTRACT

Emergence of synchronization is a remarkable collective phenomena between apparently independent agents in numerous multilevel and complex systems. The evidence of synchronization ranges from the elementary biological organisms to the most sophisticated human societies. In this paper, the problem of synchronization of nonlinearly coupled dynamical networks of Hindmarsh–Rose neurons with a sigmoidal coupling function is addressed. Sufficient condition for synchrony in terms of network structure is developed. A study on the basis of attraction of the complete synchronization is carried out for different structured networks. Also the phase synchronization of dynamical network of Hindmarsh–Rose neurons are studied. The impact of different structural properties of complex network on the phase synchronization are analyzed. The synchronization of Hindmarsh–Rose neurons are evaluated and compared on different structured network like random, regular, small-world, scale-free and modular networks. Interestingly, it was found that networks with high clustering coefficient and neutral degree mixing pattern promote better synchronization. Some chimera like state are also found in different structural networks. Further the effect of time delay dynamics on the synchronization of nonlinearly coupled network of Hindmarsh–Rose neurons are illustrated.

1. Introduction

Synchronization is a prominent collective behaviour that creates some cognition among different agents to behave cohesively. Synchronization could be observed in numerous biological, physical or social system ranging from the flash light of firefly to consensus forming in human society. Also, the function of different neuronal networks depends up on the synchronizability of individual neurons. Many brain disorders like epilepsy, Alzheimer's disease, schizophrenia and Parkinson's disease have been found to be associated to the abnormal patterns of synchronization in the brain [21,26,42]. It has been shown that the spiking, bursting, and chaotic behaviours of a single neuron could be mathematically modelled as Hindmarsh–Rose neuronal dynamics (Hindmarsh, 1987). Hence, to analyze the relation between the network topology and synchronization behaviour, in this paper, coupled Hindmarsh–Rose neurons are considered and the synchronizability of the neuronal networks are compared for different structured and random networks. In last few decades the synchronization in dynamical networks attracts the attention of several researches. To understand the synchronizability of a dynamical network the problem of determine the necessary condition for synchronization was elaborately studied. In this attempt, Pecora and Carroll [38] first defined master stability function and determine the necessary condition for synchronization.

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However, several other approaches to determine the necessary condition for synchronization has also developed in last decade. Connection graph stability method [6], matrix measure approach [12,25], linear matrix inequality [29,30], eigenvalue based approach [27,28] etc. are some remarkable development on this problem. Also Carroll and Richford studied phase space correlation of coupled Hindmarsh–Rose neurons [9]. Some previous research works focused on the effect of delay on synchronization of coupled Hindmarsh–Rose neurons. It has been shown that the delay in neuronal dynamics could provide interesting synchronization phenomena [21,22,23]. Effect of partial delay on phase synchronization is recently studied on small world network [40]. The burst synchronization of coupled Hindmarsh–Rose neurons are studied in [41,44] with different type of coupling. Stability analysis of synchronization is studied in [16,18]. Also spike phase synchronization of coupled Hindmarsh–Rose neurons are studied in [22,23].

A few research works concentrate on the effect of network topology on synchronizability. In the papers [3,31,43], it is shown that synchronizability is much better for small-world and scale-free networks, than regular networks. Also synchronization could be reduced for network with high degree heterogeneity [36]. On the contrary, Hong et al. [20] sited some examples where greater synchronizability could be achieved in for networks with high degree heterogeneity. The role of clustering coefficient on the synchronizability was also studied for some coupled dynamical networks [48] where it was predicted that increasing clustering coefficient in certain dynamical networks may reduce synchronizability. Also Hong et al. [20] have studied the impact of betweenness centrality on network synchronizability. However the relation between the network synchronizability and network properties are not very simple. A few research work also confirms that different synchronizability could be attained even for networks have the similar structural properties [1,14,17]. Some of the recent studies emphasize on synchronization transitions and effects of information transmission delay on different structured network like effect of rewiring probability on small-world neuronal networks [45] and on scale free networks [46].

The early studies on dynamical neuronal networks were based on the interconnections of neuron through electrical synapse. As result of potential difference between the neurons, an immediate physiological response is transmitted through the electrical synapse to post-synaptic side. However this electrical interaction of synapses was classically modelled as linear coupling functions. Later another type of interaction between neurons was found and now it is known that neurons can also be inter-connected via chemical synapses. In the paper [16], it has been shown that the chemical interaction between neurons could be modelled by a sigmoidal coupling function. Hence, the study of synchronization in nonlinearly coupled dynamical networks of neurons has become an important ongoing issue [12,27–29]. Also the synchronization of Hindmarsh–Rose neurons coupled via chemical as well as electrical synapse is studied as a multiplex network of neuronal coupling [24] in recent past. In recent studies it reveals that besides these some other types of coupling like field coupling can also be effective to reach phase synchronization between coupled neurons and neuronal network when synapse coupling is removed [4,18,49]. Also recent studies have shown that in case of neuronal synchronization a special case could arise where the coupled neuronal network successively moves between some high synchronous state and less synchronous state. This special type of synchronization is called chimera state. The emergence of chimera and effect of coupling topologies on it is recently studied [7]. Also the emergence of chimera states has been evident in multilayer and multiplex networks as revealed by some recent studies [32,33].

This paper is focused on exploring how structural properties of network, effect the synchronizability of the dynamical network of Hindmarsh–Rose neurons coupled by a sigmoidal coupling function. A synchronization criterion, based on eigenvalue of the Hamiltonian of outer-coupling matrix is developed to evaluated synchronizability of various network ensembles. The basin of attraction of complete synchronization states are evaluated for different structured and random networks. Also the phase synchronization phenomenon is studied for different network topology with varying different structural parameters of complex network. A comparison of synchronizability is illustrated for random, regular, small-world [47], scale-free [2] and modular [35,37] networks. The effect of varying their structural parameters on phase synchronization are also reported. Moreover the impact of some other structural properties of complex network, like clustering coefficient and assortativity, on phase synchrony are analyzed. In the entire spectrum of complex networks it is observed that high clustering coefficient and neutral degree mixing patterns or slightly disassortative mixing patterns promote better synchronization. Also some chimera like state are also found in different structural networks. Furthermore, the effect of delay on neuronal dynamics are studied on several structured networks.

2. Dynamics of coupled network of Hindmarsh–Rose neurons

The Hindmarsh–Rose model describes the dynamics of a single neuron and it could explain spiking-bursting behaviour of the membrane potential which are evident from different EEG and other brain imaging data and experiments. This model for a single neuron is computationally simple yet capable of mimicking almost all the behaviours exhibited by real biological neurons in particular the rich firing patterns. The Hindmarsh–Rose model could be written as [19]

$$\begin{aligned} x'(t) &= y - ax^3 + bx^2 - z + I \\ y'(t) &= 1 - dx^2 - y \\ z'(t) &= r(s(x + x_0) - z) \end{aligned} \tag{1}$$

Here the state variables $x(t)$ stands for membrane potential, which is represented in terms of dimensionless units. Whereas the other state variables $y(t)$ and $z(t)$, take into account the transport of ions across the membrane through the ion