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Fault tolerance of locally twisted cubes

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ABSTRACT

Let $G=(V,E)$ be a connected graph and P be graph-theoretic property. A network is often modeled by a graph $G=(V,E)$. One fundamental consideration in the design of networks is reliability. The connectivity is an important parameter to measure the fault tolerance and reliability of network. The conditional connectivity $\lambda(G, P)$ or $\kappa(G, P)$ is the minimum cardinality of a set of edges or vertices, if it exists, whose deletion disconnects G and each remaining component has property P. Let F be a vertex set or edge set of G and P be the property of with at least k components. Then we have the k-component connectivity $c\kappa_k(G)$ and the k-component edge connectivity $c\lambda_k(G)$. In this paper, we determine the k-component (edge) connectivity of locally twisted cubes LTQ_n for small k, and we also prove other properties of LTQ_n.

1. Introduction

A network is often modeled by a graph $G=(V,E)$ with the vertices representing nodes such as processors or stations, and the edges representing links between the nodes. One fundamental consideration in the design of networks is reliability [2,8]. Let $G=(V,E)$ be a connected graph, $N_G(v)$ the neighbors of a vertex v in G (simply $N(v)$), $E(v)$ the edges incident to v. Moreover, for $S\subset V$, $G[S]$ is the subgraph induced by S, $N_G(S)=\cup_{v\in S}N(v)-S$, $N_G[S]=N_G(S)\cup S$, and $G-S$ denotes the subgraph of G induced by the vertex set of $V\backslash S$. If $u,v\in V$, $d(u,v)$ denotes the length of a shortest (u,v)-path. For X, $Y\subset V$, denote by $[X, Y]$ the set of edges of G with one end in X and the other in Y. A connected graph G is called super-κ (resp. super-λ) if every minimum vertex cut (edge cut) of G is the set of neighbors of some vertex in G, that is, every minimum vertex cut (edge cut) isolates a vertex. If G is super-κ or super-λ, then $\kappa(G)=\delta(G)$ or $\lambda(G)=\delta(G)$. For graph-theoretical terminology and notation not defined here we follow [1]. All graphs considered in this paper are simple, finite and undirected.

A k-component cut of G is a set of vertices whose deletion results in a graph with at least k components. The k-component connectivity $c\kappa_k(G)$ of G is the size of the smallest k-component cut. The k-component edge connectivity $c\lambda_k(G)$ can be defined correspondingly. We can see that $c\kappa_{k+1}(G)\geq c\kappa_k(G)$ for each positive integer k. The connectivity $\kappa(G)$ is the 2-component connectivity $c\kappa_2(G)$. The k-component (edge) connectivity was introduced in [3] and [11] independently. Fábrega and Fiol introduced extraconnectivity in [4]. Let $F\subseteq V$ be a vertex set, F is called extra-cut, if $G-F$ is not connected

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and each component of $G-F$ has more than k vertices. The extraconnectivity $\kappa_k(G)$ is the cardinality of the minimum extra-cuts. Let + be the modulo 2 addition.

Definition 1. The locally twisted cube LTQ_n with 2^n vertices was introduced by Yang et al. [14]. It can be defined inductively as follows: LTQ_2 is a 4-cycle with vertex set {00, 01, 10, 11} and edge set {(00,01), (01, 11), (11,10),(10,00)}. For $n>2$, LTQ_n contains LTQ^0_{n-1} and LTQ^1_{n-1} joined according to the following rule: the vertex $u=0u_{n-2}\cdots u_0$ from LTQ^0_{n-1} and the vertex $v=1v_{n-2}\cdots v_0$ from LTQ^1_{n-1} are adjacent if and only if $v=1v_{n-2}\cdots v_0=1(u_{n-2}+u_0)u_{n-3}\cdots u_0, u_i, v_j\in\{0,1\}$.

From the definition, we can see that each vertex of LTQ_n with a leading 0 bit has exactly one neighbor with a leading 1 and vice versa. It is a n-regular graph. In fact, LTQ_n can be obtained by adding a perfect matching M between LTQ^0_{n-1} and LTQ^1_{n-1}. Hence LTQ_n can be viewed as $G(LTQ^0_{n-1}, LTQ^1_{n-1}, M)$ or $LTQ^0_{n-1}\odot LTQ^1_{n-1}$ briefly. For any vertex $u\in V(LTQ_n)$, $e_M(u)$ is the edge incident to u in M.

The locally twisted cube is an attractive alternative to hypercubes Q_n. The diameter of LTQ_n is approximately half that of Q_n. For more references, we can see [5,9,10,12–15]. In [7], Hsu et al. determined the k-component connectivity of the hypercube Q_n for $k=2,3,\ldots,n+1$. In [16], Zhao et al. determined the k-component connectivity of the hypercube Q_n for $k=n+2,n+3,\ldots,2n-4$. In [6], Guo determined the k-component (edge) connectivity of the hypercube Q_n and the folded hypercube FQ_n for small k. In this paper, we obtain that:

(1) $c\kappa_2(LTQ_n)=\kappa(LTQ_n)=n(n\geq 2)$.
(2) $c\kappa_3(LTQ_n)=2n-2(n\geq 2)$.
(3) $c\kappa_4(LTQ_n)=3n-4(n\geq 4)$.
(4) $c\lambda_2(LTQ_n)=\lambda(LTQ_n)=n$ for $n\geq 2$.
(5) $c\lambda_3(LTQ_n)=2n-1$ for $n\geq 2$.
(6) $c\lambda_4(LTQ_n)=3n-2$ for $n\geq 2$.

2. Component connectivity of locally twisted cubes

We can see that the locally twisted cube LTQ_n has important properties as follows.

Lemma 2.1. *Any two vertices of LTQ_n have at most two common neighbors for $n\geq 2$ if they have.*

Proof. By induction. If $n=2$, then the result holds. We assume that it is true for $n<k$. Suppose $n=k$ and any $u,v\in V(LTQ_n)$ such that u,v have at most two common neighbors.

If $u,v\in V(LTQ_{n-1})$, then the two common neighbors are in $V(LTQ_{n-1})$ according to inductive hypothesis. And there is not a relation between the common neighbors of u,v and the perfect matching M added to LTQ_n. Hence u,v have at most two common neighbors in LTQ_n.

By symmetry, we assume that $u\in V(LTQ^0_{n-1}), v\in V(LTQ^1_{n-1})$. The common neighbors must be obtained by adding the perfect matching M. Note that every vertex of LTQ^0_{n-1} has only one neighbor in LTQ^1_{n-1} and vice versa. Then we obtain the result. □

Corollary 2.2. *For any two vertices $x, y\in V(LTQ_n)(n\geq 2)$,*

(1) *if $d(x,y)=2$, then they have at most two common neighbors;*
(2) *if $d(x,y)\neq 2$, then they do not have common neighbors.*

Corollary 2.3. *The girth of LTQ_n is $4(n\geq 2)$.*

According to the definition of LTQ_n, if any two vertices of $V(LTQ_n)$ have only one common neighbor, then it is obtained by interchanging a pair of edges of the hypercube. Hence similar to Lemma 2.1, we have

Lemma 2.4. *Let x and y be any two vertices of $V(LTQ_n)(n\geq 4)$ such that have only two common neighbors.*

(1) *If $x\in V(LTQ^0_{n-1}), y\in V(LTQ^1_{n-1})$, then the one common neighbor is in LTQ^0_{n-1}, and the other one is in LTQ^1_{n-1}.*
(2) *If $x,y\in V(LTQ^0_{n-1})$ or $V(LTQ^1_{n-1})$, then the two common neighbors are in LTQ^0_{n-1} or LTQ^1_{n-1}.*

Lemma 2.5 *[14].* $\kappa(LTQ_n)=\lambda(LTQ_n)=n(n\geq 2)$.

Theorem 2.6. *LTQ_n is super-λ for $n\geq 3$.*

Proof. By induction. It is true for $n\leq 4$. Let $n\geq 5$. Assume that it holds for $n<k$. We will show that it is true for $n=k$.

Let $F\subseteq E(LTQ_n)$, $|F|=n$ and LTQ_n-F be not connected. Furthermore, LTQ_n-F has only two connected components. Without loss of generality, suppose $|F\cap E(LTQ^0_{n-1})|\leq\lfloor n/2\rfloor$. Then $LTQ^0_{n-1}-F$ is connected.

Note that $|[LTQ^0_{n-1}, LTQ^1_{n-1}]|=2^{n-1}>n(n\geq 5)$. If $LTQ^1_{n-1}-F$ is connected, then LTQ_n-F is connected, a contradiction.

Assume that $LTQ^1_{n-1}-F$ is not connected. We have $|F\cap E(LTQ^1_{n-1})|\geq n-1$. If $|F\cap E(LTQ^1_{n-1})|=n$, then $F\cap E(LTQ^0_{n-1})=\emptyset$ and $[LTQ^0_{n-1}, LTQ^1_{n-1}]\cap F=\emptyset$. And each vertex of $LTQ^1_{n-1}-F$ has a neighbor in $LTQ^0_{n-1}-F$, that is, LTQ_n-F is connected, a contradiction.