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Pulsatile MHD flow of a Casson fluid through a porous bifurcated arterial stenosis under periodic body acceleration

R. Ponalagusamy, S. Priyadharshini*

Department of Mathematics, National Institute of Technology, Tiruchirappalli, India

A R T I C L E I N F O

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A B S T R A C T

A mathematical model on the pulsatile flow of a Casson fluid through a porous stenosed artery with bifurcation in the presence of magnetic field and periodic body acceleration has been developed in the present study. The governing equation is expressed in terms of shear stress and the resulting momentum equation with the initial and boundary conditions is solved numerically by adopting finite difference schemes. The velocity distribution is obtained at different locations of the artery for various values of parameters involved in the study. The combined effects of bifurcation angle, stenotic height, yield stress, Hartmann number, Darcy number and time period on flow variables such as velocity, wall shear stress and resistive impedance have been observed. The shear stress along the outer wall of the parent artery is less than its corresponding value on the inner wall of the daughter artery. The shear stress along the outer wall of the parent artery and the inner wall of the daughter artery increase as Hartmann number increases. It is of interest to note that the flow resistance has a decreasing trend with the increasing value of half of the bifurcation angle and Darcy number. The wall shear stress and flow resistance are increased when the rheology of blood is changed from Newtonian to Casson fluid. It is worthwhile to note that the presence of magnetic field and porous medium increases the plug core radius which is for the first time, added to the literature. The plug core radius increases with increase in yield stress and decrease in stenotic height.

1. Introduction

The presence of a constriction or narrowing (medically termed as stenosis) in the lumen of an artery disturbs the normal blood flow and causes arterial diseases. Stenosis in the bifurcated arteries supplying blood to the brain can cause cerebral strokes, and in coronary arteries, myocardial infarction, leading to heart failure resulting in sudden death. The majority of deaths in developed countries upshot from cardiovascular diseases and most of which have been associated with some form of abnormal blood flow in bifurcated stenosed arteries. It is believed that hydrodynamic factors (e.g. wall shear stress) play a pivotal role in the development and progression of arterial stenosis. Many investigators have pointed out that the fluid dynamic and rheological properties of blood and its flow behavior through tubes with non-uniform cross-sections could play an important role in the fundamental understanding, diagnosis and treatment of many cardiovascular diseases [1–4].

* Corresponding author.
E-mail addresses: rpalagu@nitt.edu (R. Ponalagusamy), priya.nit13@gmail.com, spd.smile@gmail.com (S. Priyadharshini).

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The investigation on flow behavior of Casson fluids (polymeric solutions, fiber suspensions, food products, blood, mineral slurries, waxy crude oils, etc.) is an important subject of increasing interest among the researchers, since it has wide applications in numerous branches of engineering and medical sciences such as polymer processing industry, chemical and petroleum industry, environmental science, magnetohydrodynamics, and bio-fluid mechanics [5–7]. Lubricants play a vital role in bio-mechanics, thermal reactors and polymer industry. Yield stress of non-Newtonian fluids finds its application in bearing operations. Casson fluids possessing yield stress are used in hydrodynamics lubrication and in squeeze film bearing. The problem of a squeeze film bearing using Casson fluid as lubricant has been investigated in [8]. Barry [9] has shown that Casson fluid is used to approximate waxy crude oil flow curves. Evgeny et al. [10] analyzed the applications of Casson fluid in modeling thixotropic waxy crude oil. Casson [5] and Walwander et al. [11] have pointed out that the constitutive equation of Casson fluid model precisely matches with the flow curves corresponding to pigment suspensions in varnishes used in preparation of printing inks and in silicon suspensions. Casson fluids are used in modeling of blood oxygenators [12] and haemodialyzers [13]. Casson fluid introduces a shear-dependent viscosity consistent with the results of experiments with human blood [14] and human red blood cells in suspension [15].

The flow of blood through a stenosed artery and the effect of parameters associated with it have been investigated by Young and co-workers [1,16,17]. Several investigators [18–20] studied the concept of blood, diagnosis and treatment of several cardiovascular diseases. Blood exhibits yield stress [21,22] due to the presence of red cells in blood. The pulsatile flow of blood in a stenosed artery by modeling blood as a Casson fluid is investigated in [23,24]. Dash and Mehta [25] studied the flow characteristics of a Casson fluid in a tube filled with a homogeneous porous medium by employing the Brinkman model. In [26–28], it was suggested that the blood behaves as a Casson fluid through various experiments performed on blood varying temperature, hematocrit etc. Fung [29] analyzed the steady, fully developed and laminar flow of Casson fluids. Blair and Spanner [30] suggested that it is appropriate to assume blood as a Casson fluid through their results. Aroesty and Gross [31] studied the pulsatile flow of blood as Casson fluid through narrow uniform arteries. Since the blood behaves like an electrically conducting fluid, the magneto-hydrodynamic (MHD) principle is being adopted to decelerate the flow of blood in human arterial system and treatment of certain cardiovascular disorders. It is well known that cell separation, reduction of bleeding during surgeries and provocation of occlusion of the feeding vessels of cancer tumors are the most important applications of magnetic devices [32]. Bali and Awasthi [33] analyzed the effects of non-Newtonian nature of blood in small blood vessels by assuming blood as a Casson fluid under the influence of external magnetic field. Pulsatile flow of a Casson fluid under the influence of periodic body acceleration has been investigated in [34]. Using perturbation technique as adopted in [24], the effects of yield stress of blood and periodic body acceleration on velocity profile, wall shear stress and flow resistance are studied [35]. Akbar [36] examined the influence of magnetic field on flow of a Casson fluid in an asymmetric channel and its applications in refinement of crude oil. In these studies [23–36], the effect of bifurcation on the flow of blood has not been considered. It is well established from the physiological point of view that since the bifurcation angle at the human aortic bifurcation changes between 10 and 80 (in degrees), it is pertinent to investigate the flow phenomenon of blood by taking into account of the effect of various bifurcation angles [37].

Arteries bifurcate several times before they become capillaries. Siouffi et al. [38] investigated experimentally the unsteady flow of blood through stenosed and bifurcated artery. The pulsatile flow of blood through bifurcated artery with stenosis has been analyzed by Botwin [39]. Chakravarty and Mandal [40] developed a model of aortic bifurcation with stenosis and analyzed the pulsatile flow in the bifurcated artery by assuming blood as Newtonian fluid. The pulsatile flow of Casson fluid through a stenosed bifurcated artery and the effects of parameters such as yield stress and time on shear stress and velocity distribution have been analyzed by Shaw et al. [41].

The effects of magnetic field and porosity on physiologically and industrially important flow characteristics such as yield plane locations, plug core radius, wall shear stress and resistive impedance are not so far analyzed [38–41]. It is of interest to mention that magnetic field might be used as a flow control mechanism in medical applications. Further, the non-Newtonian nature of blood significantly influences the flow when the wall of the bifurcated arterial stenosis is considered as a porous medium. It is, therefore, pertinent to investigate the effects of magnetic field and porosity on the flow of blood through arteries.

In view of the above, the motivation of present study is to investigate the pulsatile flow behavior of blood through a porous bifurcated arterial stenosis under the influences of applied magnetic field and periodic body acceleration subject to the pulsatile pressure gradient due to normal heart action. The rheology of blood is characterized as a Casson fluid model by appropriately accounting for yield stress of blood. The combined effects of pulsatility, yield stress, stenosis, bifurcation angles, magnetic field and body acceleration, on the flow variables such as shear stress, plug core radius and flow resistance are brought out through numerical experimentation.

2. Formulation of the problem

Consider the fully developed, laminar, axially symmetric, two dimensional pulsatile flow of blood in a circular bifurcated arterial stenosis (Fig. 1). We make the following assumptions: Blood behaves as a Casson fluid, the fluid is electrically conducting and a uniform transverse magnetic field is applied along the radial direction of the artery. Let us use cylindrical polar coordinates $(\bar{r}, \bar{\theta}, \bar{z})$ to denote a material point where $\bar{r}, \bar{\theta}$ and $\bar{z}$ represent radial, circumferential and axial directions respectively.