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Scheduling with or without precedence relations on a serial-batch machine to minimize makespan and maximum cost

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ABSTRACT

In this paper, we consider several scheduling problems on a serial-batch machine for scheduling jobs with or without precedence relations. Under the serial-batch setting, the jobs in a batch are processed in succession and are removed until the last job in this batch finishes its processing. Thus, the processing time of a batch is equal to the sum of processing times of jobs in the batch. When a new batch starts, a constant setup time is required for the machine. The objectives of the problems involve minimizing makespan and a maximum cost. For these problems, we either present polynomial-time algorithms to generate all Pareto optimal points and find a corresponding Pareto optimal schedule for each Pareto optimal point, or give the strong NP-hardness proof. Experimentation results show that the proposed algorithms for the considered problems are very efficient.

1. Introduction

A serial-batch machine can simultaneously process up to b jobs as a batch (called a serial-batch), where b stands for the capacity of a batch. Concerning the capacity b, the unbounded version ($b \geq n$) and the bounded version ($b < n$) are distinguished in the literature. Under the setting of serial-batch, the processing time of a batch is equal to the sum of processing times of jobs in the batch, and the completion times of all jobs in the batch are defined to be the completion time of the batch which is the time point when the last job in the batch finishes its processing. Moreover, a constant setup time s is required for the machine at the beginning of the processing of each batch.

Suppose that a set of n jobs $\mathcal{J} = \{J_1, J_2, \ldots, J_n\}$ are given to be processed on a serial-batch machine. All jobs and the machine are available from time zero. Each job J_j $(1 \leq j \leq n)$ has a positive integer processing time p_j, a nonnegative integral due date d_j and a nonnegative integer-value cost function $f_j(t)$. For a given schedule σ, the cost function $f_j(t)$ depends on the completion time $C_j(\sigma)$ of job J_j. Suppose that all the cost functions are regular (nondecreasing in the completion times of jobs) and the value of $f_j(t)$ can be calculated in constant time for every given time $t \geq 0$. Let $C_{\max}(\sigma) = \max_{1 \leq j \leq n} C_j(\sigma)$ denote the maximum completion time (makespan), $L_{\max} = \max_{1 \leq j \leq n}(C_j - d_j)$ denote the maximum lateness, and $f_{\max}(\sigma) = \max_{1 \leq j \leq n} f_j(C_j(\sigma))$ denote the maximum cost of all jobs in schedule σ. In this paper, our

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Table 1
The time complexity of the related algorithms.

s-batch	$b \geq n$	$b < n$	$\prec$	$\preceq$	$C_{\max}$	$L_{\max}$	$f_{\max}$	Previously known	This paper
•	•					•		$O(n^2)$ [15]	
•	•						•	$O(n^5)$ [9]	$O(n^4)$
•	•				•	•		$O(n^3)$ [10]	$O(n^2)$
•	•				•		•	$O(n^5)$ [9]	$O(n^4)$
•	•		•		•		•		$O(n^4)$
•	•			•	•	•			$O(n^2)$
•		•			•	•		$O(n^6)$ [8]	$O(n^4)$
•		•	•			•			S-NP
•		•		•		•			S-NP
•		•	•		•	•			S-NP
•		•		•	•	•			S-NP

main goal is to simultaneously optimize the two objective functions $C_{\max}$ and $f_{\max}$ by enumerating all the Pareto optimal points and finding a corresponding Pareto optimal schedule for each Pareto optimal point. Such a scheduling problem is also called Pareto optimization scheduling which was described in detail in [6].

Since $C_{\max}$ and $f_{\max}$ are both regular, it allows us to only consider the schedules without idle times, and so, a schedule σ can be given by a batch sequence $(B_1, B_2, \ldots, B_l)$. For the bounded serial-batch version, a schedule $\sigma = (B_1, B_2, \ldots, B_l)$ is feasible if the number of jobs contained in each batch B_k, denoted by b_k, is no more than the batch capacity b, i.e., $b_k \leq b$. In addition, the following two types of precedence relations are involved:

- Strict precedence relations ($\prec$): if $J_i \prec J_j$, then $C_i(\sigma) < C_j(\sigma)$ in a feasible schedule σ. Equivalently, the batch including job J_i must be scheduled before the batch including job J_j, and so, J_i and J_j cannot be processed in a common batch.
- Weak precedence relations ($\preceq$): if $J_i \preceq J_j$, then $C_i(\sigma) \leq C_j(\sigma)$ in a feasible schedule σ. Equivalently, the batch including job J_i must be scheduled no later than the batch including job J_j. Note that, in this case, J_i and J_j are allowed to be processed in a common batch.

Following the notations from T'kindt and Billaut [13], the problems studied in this paper can be denoted by the following five forms:

(I) $1|\text{s-batch}, b < n|^{\#}(C_{\max}, f_{\max})$,
(II) $1|\prec, \text{ s-batch}, \ b = 2|L_{\max}$,
(III) $1|\preceq, \text{ s-batch}, \ b = 2|L_{\max}$,
(IV) $1|\prec, \text{ s-batch}, \ b \geq n|^{\#}(C_{\max}, f_{\max})$,
(V) $1|\preceq, \text{ s-batch}, \ b \geq n|^{\#}(C_{\max}, L_{\max})$,

where 's-batch' stands for the setting of serial-batch, '$b < n$' ('$b \geq n$') stands for the bounded (unbounded) version, '$b = 2$' stands for the capacity of a batch being 2, and '$\prec$' ('$\preceq$') stands for the strict (weak) precedence relations.

Batch (including serial-batch and parallel-batch) scheduling and Pareto optimization scheduling have attracted considerate research attention in the literature. We can be referred to the surveys in Allahverdi [1], Allahverdi et al. [2] and Hoogeveen [7] for detail development. For our purpose, only some related works are reviewed.

For the scheduling on an unbounded serial-batch machine, Webster and Baker [15] presented an $O(n^2)$-time dynamic programming algorithm for minimizing $L_{\max}$. Based on Webster and Baker's algorithm [15], He et al. [10] presented an $O(n^3)$-time algorithm for the Pareto optimization scheduling problem $1|\text{s-batch}, b \geq n|^{\#}(C_{\max}, L_{\max})$. Later, He et al. [9] extended this research to problem $1|\text{s-batch}, b \geq n|^{\#}(C_{\max}, f_{\max})$ and provided an $O(n^5)$-time algorithm. For the same problem, He et al. [11] presented an improved $O(n^3)$-time algorithm. However, we show in this paper that this improved $O(n^3)$-time algorithm is invalid by a counter-example presented in Section 5. Quite recently, He et al. [8] presented an $O(n^6)$-time algorithm for bounded problem $1|\text{s-batch}, b < n|^{\#}(C_{\max}, L_{\max})$.

This paper makes an additional contribution to this topic by proceeding to investigate the above mentioned five scheduling problems (I–V). Concretely, for problems (I) and (IV), we present $O(n^4)$-time algorithms, respectively. For problem (V), we present an $O(n^2)$-time algorithm. Contrastively, we prove that problems (II) and (III) are both strongly NP-hard. The summary of the results relevant to this paper is given in Table 1.

This paper is organized as follows. In Section 2, we present some preliminaries and basic lemmas. In Section 3, we present an $O(n^4)$-time algorithm for problem $1|\text{s-batch}, b < n|^{\#}(C_{\max}, f_{\max})$. In Section 4, we present the strong NP-hardness proof for problems $1|\prec, \text{ s-batch}, \ b = 2|L_{\max}$ and $1|\preceq, \text{ s-batch}, \ b = 2|L_{\max}$. In Section 5, we present an $O(n^4)$-time algorithm for problem $1|\prec, \text{ s-batch}, \ b \geq n|^{\#}(C_{\max}, f_{\max})$ and show that the $O(n^3)$-time algorithm for problem $1|\text{s-batch}, b \geq n|^{\#}(C_{\max}, f_{\max})$ presented in He et al. [11] is invalid. In Section 6, we present an $O(n^2)$-time algorithm for problem $1|\preceq, \text{ s-batch}, \ b \geq n|^{\#}(C_{\max}, L_{\max})$. The results of some computational experiments are provided to support our proposed algorithms in the final section.