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Strong behavioral similarities in timed-arc Petri nets☆

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ABSTRACT

The concept of process similarity has attracted the attention of many researchers in the recent literature, since it measures the degree of proximity of processes. Similarities are therefore useful for the efficient management of large repositories, since they allow us to find the appropriate process models among hundreds or thousands of possible candidates. However, time constraints are usually omitted when these similarity measures are defined, so in this paper two similarity measures are defined over a timed extension of Petri nets, the so-called timed-arc Petri nets, in which tokens are assigned an age indicating the time elapsed from creation, and PT-arcs (place to transition arcs) are labeled with time intervals that are used to restrict the age of the tokens that can be used to fire the adjacent transition.

1. Introduction

A similarity measure provides us with a quantitative information about the degree of proximity in the behavior of two processes. This has been a focus of recent research since process models are extensively used in many areas of current technologies, such as software development, enterprise architecture or product life cycle management. Thus, as Rosemann states [21], multinational corporations face the problem of managing collections of thousands of process models, and similarity measures are an important tool in finding the process model whose structure or behavior is closest to a specific problem. Similarities can therefore be used for the efficient management of large repositories, as Wang [26] states, since they allow us to find the appropriate process models among hundreds or thousands of possible candidates.

Petri nets are a useful formalism for describing and analyzing process models, because of their graphical nature and the solid mathematical foundations supporting them. Several timed extensions of the basic model have been proposed to expand their areas of application to those systems which exhibit time-dependent behavior that should be considered both in the modeling and the analysis process, such as distributed systems, communication systems and real-time systems. Timed business processes can therefore be modeled and analyzed with timed Petri nets.

Timed-Arc Petri Nets (TAPNs) [7,15] are a timed extension of Petri nets in which tokens are assigned a natural value indicating the time elapsed from creation (their *age*), and arcs from places to transitions are labeled with time intervals that establish restrictions on the age of the tokens that can be used to fire the adjacent transitions. Thus, a transition can be fired when all its precondition places contain at least one token with an age in the time interval of the arc connecting the place with the transition. When the transition is fired, one token fulfilling this condition is removed from each precondition place, and one token of age 0 is produced in each postcondition place.

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In this paper, two timed behavioral similarity measures over bounded Marked Labeled Timed-Arc Petri Nets (MLTAPNs) are introduced. They are called *btts-similarity* and *lcp-similarity*, and are defined by using a reduced version of the timed reachability graphs of the given MLTAPNs.

The main drawback of this approach is the state space explosion, which is due to the enormous number of states these MLTAPNs will have in general. But the state graph size is not the only problem for the computation of timed behavioral similarities, since their computation is based on a comparison between the timed occurrence sequences of the given MLTAPNs, and obviously there will be, in general infinitely many of these sequences. The technique used in this paper is based on two timed versions of the longest common prefixes of the timed transition sequences of the MLTAPNs under consideration, but in contrast to other approaches loops are processed by considering a finite number of repetitions of them, which allows us to obtain a better approximation in repetitive behaviors.

This paper is structured as follows. The related work is presented in Section 2. Labeled Timed-Arc Petri Nets (TAPNs) and their semantics are formally defined in Section 3. Timed reachability graphs for marked TAPNS and a reduced version of them is defined in Section 4. The general concept of similarity and two timed similarity proposals are presented in Section 5. The evaluation of both similarities by experimental results is described in Section 6. Finally, the conclusions and plans for future work are presented in Section 7.

2. Related work

A survey of the different approaches to introducing time into Petri nets was carried out by F.D.J. Bowden [8]. We can identify a first group of models, which assign time delays to transitions, either using a fixed and deterministic value [20] or choosing it from a probability distribution [3]. Other models use time intervals to establish the enabling times of transitions [18]. Finally, there are also some models that introduce time on tokens [1,7,15]. In such a case, tokens are classified into two different classes: available and unavailable. Available tokens are those that can be immediately used for firing a transition, while unavailable tokens cannot. We have to wait for a certain period of time for these tokens to become available, although it is also possible for a token to remain unavailable forever (such tokens are said to be *dead*). Cerone and Maggiolo-Schettini [9] have defined a very general model (statically timed Petri nets), in which timing constraints are intervals that are statically associated with places, transitions and arcs. Thus, models with timing constraints attached only to places, transitions or arcs can be obtained as particular subclasses of this general framework.

In this paper, we consider Timed-Arc Petri Nets (TAPNs) [7,15], a timed extension of Petri nets in which tokens are assigned a natural value indicating the time elapsed from creation (their *age*), and arcs from places to transitions are also labeled with time intervals, which establish restrictions on the age of the tokens that can be used to fire the adjacent transitions. Timed-arc Petri nets are supported by some existing tools. TPAL [25] is a tool that allows us to obtain TAPN models from algebraic descriptions written in the TPAL language [24], and TAPAAL [11] is a tool that supports TAPNs extended with age invariants and both inhibitor and transport arcs, and which is also equipped with an efficient verification engine.

This is a rather pathological model, as reachability is undecidable [22], even in the case where tokens in different places are not required to age at the same rate [19], whereas some other known properties of Petri nets, such as boundedness, coverability and even termination are decidable [2,5,12,16]. The detection of *dead transitions*, namely those that can never be fired, is also decidable [23] for this model.

As mentioned above, in this paper we define two timed behavioral similarity measures over bounded Labeled Timed-Arc Petri Nets (LTAPNs), in contrast to the structural approaches [10,13,17], which define measures on the basis of the model structure, disregarding its dynamic behavior. An intermediate approach has been taken by Bae et al. [6], where workflow dependency graphs are used to obtain normalized process network matrices, from which process similarity measures are computed as metric space distances between these normalized matrices. An untimed behavioral similarity measure based on transition adjacency relations in workflow nets has been defined in [28]. In this case, sequential firing of pairs of transitions is exploited in order to compute the TAR-similarity measure. Another untimed similarity measure has been defined by Wang et al. [26], in which principal transition sequences (PTS) are derived from the coverability tree of a Labeled Petri Net (LPN). These PTS are a characterization of the (possibly infinite) set of transition sequences of the LPN and are used to compute a similarity measure on the basis of their longest common subsequences. Wang et al. [27] have also defined a similarity measure for untimed Petri nets by using the edit distance between node labeled versions of the coverability graphs. A different line of work exploits the observed behavior of the systems by using event logs: Alves de Medeiros et al. [4] have defined a process trace equivalence measure based on the observed behavior, using event logs to capture their typical behavior. Finally, a survey of existing methods for performing a comparative graph analysis can be found in [14], in which the authors make a distinction between methods for deterministic and random graphs.

3. Labeled timed-arc Petri nets

Labeled Timed-Arc Petri Nets (LTAPNs) have their transitions labeled with actions, their tokens are annotated with an age (an integer value indicating the time elapsed from creation), and the arcs connecting places with transitions are assigned a time interval which limits the age of the tokens that can be used to fire the adjacent transition.

However, a transition is not forced to fire when all its precondition places contain tokens with an adequate age, and the same is true even if the age of any of these tokens is about to expire. It is therefore possible that some tokens become *dead*,