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A fast and robust sub-optimal control approach using reduced order model adaptation techniques

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ABSTRACT

Classical adjoint-based optimization approach for the optimal control of partial differential equations is known to require a large amount of CPU time and memory storage. In this article, in order to reduce these requirements, a posteriori and a priori model order reduction techniques such as POD (Proper Orthogonal Decomposition) and PGD (Proper Generalized Decomposition) are used. As a matter of fact, these techniques allows a fast access to the temporal dynamics of a solution approximated in a suitable subspace of low dimension, spanned by a set of basis functions that form a reduced basis. The costly high fidelity model is then projected onto this basis and results in a system of ordinary differential equations which can be solved in quasi-real time. A disadvantage of considering a fixed POD basis in a suboptimal control loop, is basically the dependence of such bases on a posteriori information coming from high fidelity simulations. Therefore, a non robustness of the POD basis can be expected for certain perturbations in the original parameter for which it was built. As a result, update the reduced bases with respect to each variation in the control parameter using the POD method is still costly. To get over this difficulty, we equip the usual reduced optimal control algorithm with an intermediate basis adaptation step. The first proposed approach consists in adapting the reduced basis for a new control parameter by interpolating over a set of POD bases previously computed for a range of control parameters. To achieve that, an interpolation technique based on properties of the tangent subspace of the Grassmann manifold (ITSGM) is considered. The second approach is the PGD method, which by nature, enrich a space time decomposition trying to enhance the approximation by learning from its own errors. Relaying on this property, this method is employed in the control loop as a basis corrector, in such a way the given spatial basis is adapted for the new control parameter by performing just few enrichments. These two approaches are applied in the sub-control of the two dimensional non-linear reaction-diffusion equations and Burgers equations.

1. Introduction

Optimal control of partial differential equations (PDE) using the adjoint-based approach requires to solve many times the state and their associated adjoint equations, which results in very large computational time and important memory storage. Therefore it is not possible to consider doing real-time or quasi-real time control by using conventional resolution tech-

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niques based for example on finite element, finite volume or finite difference methods. In this scope, model order reduction techniques are attracting increasing interest due to their ability to represent a solution of any dynamical system in a low dimensional subspace, resulting by consequence in a fast access to the temporal dynamics of the problem. Reducing the order of a PDE requires first the construction of a representative reduced spatial basis. Then, by the Galerkin projection of the PDE onto this basis, a system of ordinary differential equations of low order, called through this paper reduced order model (ROM), is obtained. Finally, the approximation of the PDE solution over the time interval is achieved trough the ROM equation which is very fast to solve. Among existing model reduction methods, the POD has received much attention over the past two decades and is still the most widely used. Since the POD method requires a set of snapshots, often obtained from solving the high fidelity model, the POD basis is not robust when the parameter rolls away the original parameter for which it was built. This is a blocking aspect, especially inside an optimal control loop where the control parameter vary.

Initial studies in the context of sub-optimal control using the POD method have been carried out, more particularly for fluid mechanics applications. For instance Bergmann et al. [1] controlled the time angular velocity of a rotating cylinder to optimize the drag of an incompressible viscous flow past a circular cylinder. Ravindran [2] considered the control problem of reducing recirculation behind the step of a flow in a backward facing step channel, by acting on the movement of a wall portion or through blowing on it. In these two studies the POD basis was chosen once and fixed all along the optimization process. Nevertheless, this hypothesis is strong and assumes that this chosen POD basis is able to approximate quite accurately the solutions associated parameters which will be tracked by the optimization algorithm. The fact that a POD basis is optimal only for the parameter for which it was built make this hypothesis suspect in general. An ensemble of snapshots sets obtained form a set of different parameters can be used to generate a basis with a wider region of trust . This approach was used for instance by Tallet et al. [3] in the control of the anisothermal Navier–Stokes equations in a differential heated lid driven cavity, by acting on the velocity and temperature on the walls. In the following this approach is named Multiple Parametrized Snapshots method (MPS).

In this paper, we propose an efficient and fast sub-optimal control algorithm with a basis adaptation step. Two approaches are considered for adaptation. The first approach consists in generating a set of POD bases for different parameters, and using a special method of interpolation based on properties of the tangent subspace of the Grassmann manifold (ITSGM) to obtain the basis associated to a given parameter. This approach was firstly introduced by Amsallem and Farhat [4] in the context of aeroelasticity. It involves notions of a geodesic path between two points, and of the tangent subspace at a point on the Grassmann manifold. The second approach consists in a correction process of the basis at each iteration of the control algorithm by using the so called PGD method. This method originally based on a space-time separation of the solution was proposed by Ladevéze et al [5–7] under the name of radial approximation to solve nonlinear structural mechanics. This space-time representation was also used in [8,9] to solve stochastic problems; in this context, the method is named Generalized Spectral Decomposition. Ammar et al. extended the PGD to multidimensional decomposition to solve models of polymeric systems [10,11]. Since these works, the PGD was applied to efficiently solve many problems of heat transfer [12], fluid mechanics [13–17], quantum chemistry [18], solid mechanics [19]... A review of the method can be found in [20]. In the context of optimal control and optimization, the PGD had already been investigated in previous works [21–24]. In these studies the PGD is used to build a general separated form of the solution for a large amount of parameters. This computational vade-mecum is afterwards used in order to perform real-time optimal control. In the present work, the considered approach is different and does not require any computations beforehand. More specifically, the PGD is used here to enrich time-space separated functions in order to enhance their accuracy. This process is embedded in the optimal control loop to operate whenever is needed like a basis corrector.

The paper is organized as follows : The 2nd section introduces the general optimal control problem and highlights the resolution difficulties faced with the classical adjoint based approach. In the 3rd section, the reduced order model approach using the Galerkin projection is presented. The 4th and 5th sections provide the details of calculations of POD and MPS-POD bases. The bases adaptation approaches ITSGM and PGD are exposed in Sections 6 and 7, respectively. The proposed reduced control approach with the basis adaptation step is afterwards outlined in Section 8. In Sections 9 and 10, the reduced control approach using ITSGM and PGD is numerically investigated in the examples of two dimensional reaction-diffusion and Burgers equations. Finally, some conclusions and future works are presented.

2. General optimal control problem formulation

Let K be a closed convex subset of a Banach space Z and V and W two Hilbert spaces endowed respectively with the inner products $\langle \cdot, \cdot \rangle_V$ and $\langle \cdot, \cdot \rangle_W$. The subject of investigation in the present paper is the non-linear variational problems of the form

$$\min \mathcal{J}(u, \omega) \text{ over } (u, \omega) \in V \times K \tag{1}$$

subject to

$$\mathcal{N}(u, \omega) = 0 \tag{2}$$

where $\mathcal{J} : V \times K \to \mathbb{R}$ denotes the cost functional, $u \in V$ the state variable and $\omega \in K$ the control parameter. The relationship between the state and control variables is described by the constraints map $\mathcal{N} : V \times Z \to W$. This relationship can typically be governed by a partial differential equation evolving in a time interval $]0, T]$, $T > 0$, and defined in a domain space $\Omega \subset \mathbb{R}^d$