Applied Mathematics and Computation 333 (2018) 443–459

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Nonlinear bending of elastoplastic functionally graded ceramic-metal beams subjected to nonuniform distributed loads

Dinh Kien Nguyen[a,b,*], Khoa Viet Nguyen[a], Van Manh Dinh[a], Buntara S. Gan[c], Sergei Alexandrov[d]

[a] *Institute of Mechanics, VAST, 18 Hoang Quoc Viet, Hanoi, Vietnam*
[b] *Graduate University of Science and Technology, VAST, 18 Hoang Quoc Viet, Hanoi, Vietnam*
[c] *Department of Architecture, College of Engineering, Nihon University, Koriyama, Fukushima-ken 963–8642 Japan*
[d] *Laboratory of Fracture Mechanics, Institute for Problems in Mechanics, Moscow 11926, Russia*

ARTICLE INFO

Keywords:
Functionally graded beam
Nonlinear bending
Elastoplastic material
Finite element method

ABSTRACT

Nonlinear bending of elastoplastic functionally graded (FG) ceramic-metal beams subjected to nonuniform distributed loads is investigated by the finite element method. A bilinear stress-strain relation with isotropic hardening is assumed for elastoplastic behavior of metal, and the elastoplastic properties of the FG ceramic-metal material are evaluated by using Tamura–Tomota–Ozawa (TTO) model. Based on Euler–Bernoulli beam theory, a nonlinear finite element formulation, taking the effect of plastic deformation into account, is derived and used in the investigation. The formulation employing nonlinear von Kámán strain-displacement relationship is derived by using the physical neutral surface as reference plane. An incremental-iterative procedure based on Newton–Raphson method, in which the plastic equation is solved at Gauss points for updating the stress and evaluating the element formulation, is employed to solve nonlinear equilibrium equations. The elastoplastic behavior is illustrated for a FG beam composed of TiB and Ti. The numerical results show that yielding in the FG beam occurs at the layer near the ceramic surface earlier than it does at the layer near the metal surface. The effect of the material distribution, plastic deformation on the nonlinear behavior of the beam with various end conditions is investigated in detail. The formation and propagation of plastic zone inside the beam during the loading process is also examined and highlighted.

1. Introduction

Functionally graded materials (FGMs) have received great interest from researchers since they were first initiated by Japanese scientists in 1984 [1]. FGMs are produced by varying gradually volume fraction of constituent materials, usually ceramics and metals, in one or more desired spatial directions. The effective properties of the resulted FGM exhibit continuous change, and thus eliminating interface problems and reducing stress concentrations which often seen in traditional laminate or fiber-reinforce composite. FGMs have promising applications in aerospace, transportation, nuclear engineering,

* Corresponding author at: Institute of Mechanics, VAST, 18 Hoang Quoc Viet, Hanoi, Vietnam.
E-mail address: ndkien@imech.vast.vn (D.K. Nguyen).

https://doi.org/10.1016/j.amc.2018.03.100

electronics, and biomedical engineering [2]. A large number of investigations on the analysis of functionally graded (FG) structures subjected to different loadings using both analytical and numerical methods was summarized by Birman and Byrd [3], contributions that are most relevant to the present work are briefly discussed below.

Praveen and Reddy [4] derived a plate element for studying the nonlinear static and dynamic behavior of FG ceramic-metal plates. A first-order finite beam element for the thermo-elastic analysis of FG beams was formulated by Chakraborty et al. [5]. The element using the solution of the governing static equations of a FG beam segment to interpolate the displacement field is high accurate and efficient. Based on the concept of effective principle axes, Li et al. [6] studied the stress distribution of FG beams with rectangular cross-section. Kadoli et al. [7] adopted the third-order shear deformation beam theory in their derivation of finite element formulations for studying the static behavior of FG beams under ambient temperature. Kang and Li [8,9] considered the shift of the neutral axis in their derivation of the large displacement solutions for nonlinear cantilever FG beams subjected to a transverse tip load or a tip moment. Lee et al. [10] presented a finite element procedure for investigating the post-buckling response of FG plates under a combination of compressive and thermal loads. Using an analytical method, Huang and Li [11] studied the bucking of axially graded Bernoulli columns with varying cross sections. Using the finite element method, Alshorbagy et al. [12] computed the natural frequencies of FG Euler–Bernoulli beams. Shahba et al. [13] employed the exact shape functions of a uniform shear deformation beam in their derivation of a finite element formulation for studying the free vibration of a tapered Timoshenko beam made of axially FGM. The exact variation of the section profile was used in evaluating the element formulation for improving the accuracy and efficiency of the numerical results. Based on a total Lagrangian formulation, Almeida et al. [14] studied the geometrically nonlinear behavior of FG beams subjected to end forces. Using the physical neutral surface as reference plane, Taeprasartsit [15] derived the displacement functions and buckling loads of perfect and imperfect Euler–Bernoulli FG columns. Also using the neutral surface as reference plane, Levyakov [16] derived the elastica solution for FG beam under a thermal loading. Based on the third-order shear deformation beam theory, Zhang [17] derived the constitutive equations for studying the nonlinear bending of FG beams. Eltaher et al. [18] derived a beam finite element for studying the free vibration of FG Euler–Bernoulli macro/nano beams. Nguyen [19,20], Nguyen and Gan [21] studied the large deflection of nonuniform FG beams by using the co-rotational finite element formulations. Niknam et al. [22] presented an analytical method for investigating the nonlinear bending of tapered FG beams subjected to thermal and mechanical loading. Based on the higher-order shear deformation beam theory, Frikha et al. [23] proposed a two-node C^0 beam element for static analysis of FG beams. The beam element using a mixed formulation for improvement of the shear field is capable to give accurate deflections and stresses of FG cantilever and simply supported beams. In [24], Nguyen and Tran developed a fast convergent co-rotational finite element formulation for the large displacement analysis of FG sandwich beam and frame structures. The element based on Euler–Bernoulli beam theory is derived using the exact solution of nonlinear equations of a beam segment to interpolate the axial and transverse displacements.

Analysis of elastoplastic FG structures has been drawn considerable attention from researchers in recent years. In this line of works, a composite model proposed by Tamura el al. [25] (referred to as TTO model herein) is widely adopted in evaluating effective elasoplastic properties of FGM. Based on the TTO model, Gunes et al. [26] employed the finite element code LS-DYNA to study the elastoplastic response of FG circular plates under low-velocity impact loads. Jahromi et al. [27] employed a bilinear tress-strain relationship in modeling the elastoplastic behavior of a FG rotating disk. The stress field of the disk is then computed with the aid of the commercial finite element package ABAQUS. Nie and Zhong [28] derived the solutions for stress distribution of curved elastoplastic FG beams subjected to pure bending. Huang and Han [29], Huang et al. [30] adopted a multi-linear hardening elastoplastic material to study the elastoplastic buckling of FG cylindrical shells subjected to axial and torsion loads, respectively. Also using the multi-linear hardening elastoplastic material model, Zhang et al. [31] studied the buckling behavior of elastoplastic FG cylindrical shells under a combination of the axial compressive load and external pressure. With the aid of Galerkin method, a detail examination on the effects of dimensional parameters and elastoplastic material properties on the stability region and elastoplastic interface of the shells has been carried out in [31]. The finite element method was employed by Trinh et al. [32] in studying the post-buckling of FG beams resting on a nonlinear elastic foundation. The authors showed that the post-buckling behavior and limit load of the beams are significantly influenced by the plastic deformation.

In this paper, the nonlinear bending of elastoplastic FG ceramic-metal beams subjected to various types of nonuniform distributed load is studied by the finite element method. To this end, a nonlinear finite element formulation based on Euler–Bernoulli beam theory is derived and employed in the study. A bilinear stress-strain model with isotropic hardening is assumed for the metal phase, and the TTO model is adopted to evaluate the effective elastoplastic properties of the FG ceramic-metal material. The formulation adopted nonlinear von Kámán strain-displacement relation is derived by using the physical neutral surface of the beam as reference plane. It should be noted that for FG beams considered herein, both the yield stress and tangent modulus vary in the beam thickness, and thus the non-layer method used in analysis of elastoplastic homogeneous beams by defining the yield criterion through the internal forces [33] is unable to employ for FG beams. Therefore, the layer beam approach in which the plastic equation is solved at quadrature Gauss points is employed in updating the axial stress and evaluating the element nodal force vector and tangent stiffness matrix, is necessary to use. Based on the derived finite element formulation, the nonlinear equilibrium equations for the beam are constructed and solved by an incremental-iterative procedure based on Newton–Raphson method. The elastoplastic response is illustrated for a FG TiB-Ti beam with various end conditions. The effect of material properties, plastic deformation on the response of