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Two-step shock waves propagation for isothermal Euler equations

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ABSTRACT

The feedback control algorithm is applied to provide stable propagation of a two-step shock waves for nonlinear isothermal Euler equations despite the desired profile and velocity of the waves do not correspond to an analytical solution of the equations. Two cases are considered: transition to the two-step shock wave solution form the usual one-step wave and generation of a wave with a two-step front from an initially undisturbed velocity field. In both cases arising of two-step shock waves is obtained and an influence of the control algorithm coefficients on the shape of the waves is established.

1. Introduction

The solutions to nonlinear coupled wave equations contain restrictions concerning their shapes, velocities and initial phases. Only particular solutions of nonintegrable nonlinear wave equations may be obtained by direct methods, see, first of all, [1–3], however, only a few works are devoted to finding exact solutions to the coupled equations [4–7]. Even small variations in the relative positions of the initial coupled variables give rise to significant variations in the following waves behavior: loosing of the wave localization, instability, variation in the polarity of the wave and even change of the type of the wave, say, from a bell-shaped to a kink-shaped wave, see, e.g., [8,9].

The feedback control can support stable solution, in particular, oscillations caused by imperfect initial conditions can be suppressed by an inclusion of the additional control terms in the equation. This is provided by an additional control term in the equation using the speed-gradient control approach [10–12]. Recently, the speed-gradient control has been extended by single nonlinear wave equations [13–15].

The Euler equations attract considerable attention due to their use in the gas dynamics problems [16–20]. Despite some analytical solutions, see, e.g., [21], these equations are mainly solved numerically, besides above mentioned references, see also [19,20,22–24]. Among their solutions the shock waves are one of the most important, in particular, when they contain several steps [20].

This paper deals with the control of the stable two-step shock waves propagation of the coupled isothermal nonlinear Euler equations. Its traveling monotonic one-step shock wave solution can be obtained, as suggested in Section 1, as a limiting case of the isothermal Navier–Stokes equations. However, a two-step shock wave solution is not described analytically.

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The feedback control algorithm is developed in Section 2 to support its stale propagation. It is shown numerically that various sets of initial conditions may result into a two-step shock wave propagation for the both coupled variables of the Euler equations. Moreover, the control also suppresses scheme dispersion of the Lax–Wendroff scheme used for calculations that gives rise to the absence of oscillations on the profile of the shock waves.

2. Isothermal Euler equations

The isothermal coupled compressible Euler equations are

$$\rho_t + (\rho\ u)_x \ = \ 0, \tag{1}$$

$$(\rho\ u)_t + (\rho\ u^2 + a^2\ \rho)_x \ = \ 0, \tag{2}$$

where ρ is density, u is velocity, a is constant sound speed. Using the notation $v\ =\ \rho u$, the equations are conveniently rewritten in the form

$$F_t + U_x \ = \ 0, \tag{3}$$

where

$$F = \begin{pmatrix} \rho \\ v \end{pmatrix}, \quad U = \begin{pmatrix} v \\ \frac{v^2}{\rho} + a^2 \rho \end{pmatrix}$$

Analytical solutions may be obtained for the boundary conditions in the form

$$\rho \ \to \ \rho_{\pm\infty} \quad \text{at} \ \ x \ \to \ \pm\infty, \tag{4}$$

$$v \ \to \ 0 \ \text{ at } \ x \ \to \ \infty, \ v \ \to \ v_{-\infty} \quad \text{at} \ \ x \ \to \ -\infty. \tag{5}$$

The isothermal Navier–Stokes equations arise as a generalization of Eqs. (1) and (2) by adding the viscosity term $\nu(\rho u_x)_x$, ν is kinematic viscosity coefficient, in Eq. (2). In the notations ρ, v, the Navier–Stokes equations are

$$\rho_t + v_x \ = \ 0, \tag{6}$$

$$v_t + \left(\frac{v^2}{\rho} + a^2 \rho - \nu\ \rho \left[\frac{v}{\rho} \right]_x \right)_x \ = \ 0, \tag{7}$$

The traveling shock wave solution to Eqs. (6) and (7) depends on the phase variable $\theta\ =\ x - V\ t$. Then Eq. (6) is integrated once providing the solution for ρ,

$$\rho \ = \ \frac{v}{V} + \rho_\infty. \tag{8}$$

The solution (8) satisfies the boundary condition (4) at $x\ \to\ -\infty$ while another boundary condition defines the phase velocity V,

$$V \ = \ \frac{v_{-\infty}}{\rho_{-\infty} - \rho_\infty}. \tag{9}$$

Substitution of Eqs. (8) and (9) into Eq. (7) results in the ordinary differential equation (ODE) for v,

$$\left(a^2 - \frac{v_{-\infty}^2}{(\rho_{-\infty} - \rho_\infty)^2} \right) v + \frac{a^2}{v_{-\infty}(\rho_{-\infty} - \rho_\infty)}\ v^2 - \frac{\nu\ \rho_\infty v_{-\infty}}{\rho_{-\infty} - \rho_\infty}\ v_\theta \ = \ 0. \tag{10}$$

This equation corresponds to once integrated ODE reduction of the Burgers equation. Its known kink-shaped solution or a shock wave solution,

$$v \ = \ \frac{v_{-\infty}}{2}\ (1 - \tanh[k\ (x - V\ t - x_0)]), \tag{11}$$

exists provided that

$$k \ = \ \frac{(\rho_{-\infty} - \rho_\infty) a}{2\ \nu \sqrt{\rho_{-\infty}\rho_\infty}}, \ \ v_{-\infty} \ = \ \frac{(\rho_{-\infty} - \rho_\infty) a \sqrt{\rho_{-\infty}}}{\sqrt{\rho_\infty}}. \tag{12}$$

The last expression in Eqs. (12) gives rise to the final expression for the phase velocity V,

$$V \ = \ a \sqrt{\frac{\rho_{-\infty}}{\rho_\infty}} \tag{13}$$