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Graph operations based on using distance-based graph entropies

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ABSTRACT

Let G be a connected graph. The eccentricity of vertex v is the maximum distance between v and other vertices of G. In this paper, we study a new version of graph entropy based on eccentricity of vertices of G. In continuing, we study this graph entropy for some classes of graph operations. Finally, we compute the graph entropy of two classes of fullerene graphs.

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1. Introduction

All graphs considered in this paper are simple, connected and finite. Let x and y be two arbitrary vertices of graph G, the distance between them is the length of the shortest path connecting them denoted by $d(x, y)$.

Let Γ be a group and Ω be a non-empty set. An action of group Γ on the set Ω is a function φ: $\Gamma \times \Omega \to \Omega$, where $(g, x) \to \varphi(g, x)$ that satisfies in the following two properties (we denote $\varphi(g, x)$ as x^g): $\alpha^e = \alpha$ for all α in Ω and $(\alpha^g)^h = \alpha^{gh}$ for all g, h in Γ. The orbit of an element $\alpha \in \Omega$ is denoted by α^G and it is defined by the set of all $\alpha^g, g \in G$.

A bijection f on the vertices of graph X is called an automorphism of X which preserves the edge set E. In other words, the bijection f on $V(X)$ is an automorphism if and only if $e = uv$ is an edge, then $f(u)f(v)$ is an edge of E (which the image of vertex u is denoted by $f(u)$). The set of all automorphisms of X denoted by $Aut(X)$ forms a group under the composition of mappings. This group acts transitively on the set of vertices, if for a pair of vertices such as u and v in $V(X)$, there is an automorphism $g \in Aut(X)$ such that $g(u) = v$. In this case, we say that X is vertex-transitive. An edge-transitive graph can be defined similarly.

Here in the next section, we state necessary definitions and some preliminary results. Section 3 contains the main results of this paper namely the computation of the entropy of some product graphs. Finally, we determine the entropy of two infinite classes of fullerene graphs in Section 4.

2. Preliminaries

In this paper, the symbol *log* denotes the logarithm based on the basis 2. In the following, we introduce entropy measures based on Shannon's entropy, see [21].

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Let $p = (p_1, \ldots, p_n)$ be a probability vector, namely $0 \le p_i \le 1$ and $\sum_{i=1}^{n} p_i = 1$. Shannon's entropy of p is defined as

$$I(p) = -\sum_{i=1}^{n} p_i log(p_i).$$

To define information-theoretic graph measures, we often consider a tuple $(\lambda_1, \ldots, \lambda_n)$ of non-negative integers $\lambda_i \in \mathbb{N}$. This tuple forms a probability distribution $p = (p_1, \ldots, p_n)$, where [6–9]

$$p_i = \frac{\lambda_i}{\sum_{j=1}^{n} \lambda_j}.$$

Therefore the entropy of tuple $(\lambda_1, \ldots, \lambda_n)$ is given by

$$I(\lambda_1, \ldots, \lambda_n) = -\sum_{i=1}^{n} p_i log(p_i) = log(\sum_{i=1}^{n} \lambda_i) - \sum_{i=1}^{n} \frac{\lambda_i}{\sum_{j=1}^{n} \lambda_j} log(\lambda_i).$$

In the literature, there are various ways to obtain the tuple $(\lambda_1, \ldots, \lambda_n)$ like the so-called magnitude-based information measures introduced by Bonchev and Trinajstić [2], or partition-independent graph entropies, introduced by Dehmer which are based on information functionals. We are now ready to define the entropy of a graph due to Dehmer and co-workers [10–12] by using special information functionals. Let $G = (V, E)$ be a undirected connected graph. For a vertex $v_i \in V$, we define

$$p(v_i) = \frac{f(v_i)}{\sum_{j=1}^{|V|} f(v_j)},$$

where f represents an arbitrary information functional. Observe that

$$\sum_{j=1}^{|V|} p(v_j) = 1.$$

Hence we can interpret the quantities $p(v_i)$ as vertex probabilities. A definition of graph entropy of graph G can be derived immediately as follows. Let $G = (V, E)$ be a undirected connected graph and f be an arbitrary information functional. The entropy of G is defined as

$$\begin{aligned} I_f(G) &= -\sum_{i=1}^{|V|} \frac{f(v_i)}{\sum_{j=1}^{|V|} f(v_j)} log\left(\frac{f(v_i)}{\sum_{j=1}^{|V|} f(v_j)} \right) \\ &= log\left(\sum_{i=1}^{|V|} f(v_i) \right) - \sum_{i=1}^{|V|} \frac{f(v_i)}{\sum_{j=1}^{|V|} f(v_j)} log(f(v_i)). \end{aligned}$$

There exists a large number of information functionals which can be used to characterize a graph by using entropy: one of them is based on eccentricity. The eccentricity of vertex v is $\sigma(v) = max_{u \in V} d(u, v)$. Let $G = (V, E)$, for a vertex $v_i \in V$, we define f as $f(v_i) := c_i \sigma(v_i)$ where $c_i > 0$ for $1 \le i \le n$. The entropy based on f denoted by $If_\sigma(G)$ is defined as follows:

$$If_\sigma(G) = log\left(\sum_{i=1}^{n} c_i \sigma(v_i) \right) - \sum_{i=1}^{n} \frac{c_i \sigma(v_i)}{\sum_{j=1}^{n} c_j \sigma(v_j)} log(c_i \sigma(v_i)). \quad (1)$$

In addition, if $c_i s$ are equal, then

$$If_\sigma(G) = log\left(\sum_{i=1}^{n} \sigma(v_i) \right) - \sum_{i=1}^{n} \frac{\sigma(v_i)}{\sum_{j=1}^{n} \sigma(v_j)} log(\sigma(v_i)). \quad (2)$$

For further information, see [3–5,16,18,20].

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