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Static output feedback set stabilization for context-sensitive probabilistic Boolean control networks

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ABSTRACT

In this paper, we investigate the static output feedback set stabilization for context-sensitive probabilistic Boolean control networks (CS-PBCNs) via the semi-tensor product of matrices. An algorithm for finding the largest control invariant set with probability one is obtained by the algebraic representations of logical dynamics. Based on the analysis of the set stabilization, necessary and sufficient conditions for *S*-stabilization are obtained. Static output feedback controllers are designed to achieve *S*-stabilization for a CS-PBCN. At last, examples to study metastatic melanoma are given to show the effectiveness of our main results.

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1. Introduction

Boolean networks (BNs) [1] as a valuable tool for bio-genetic engineering, which has attracted the interest of many experts and scholars. And it has been used for simulating, analyzing and reaching the genetic regulatory networks [2]. In BNs, 1 (or 0) corresponds to the on (or off) state of the Boolean variable. And the update of each state for the Boolean variable is governed by logical relationship at every discrete time. When referring to the Boolean control networks (BCNs), it can be seen as BNs added controllers. As for the research of BNs and BCNs, Cheng first proposed semi-tensor product (STP) in [3] and [4]. Using STP, a BN can be converted into a standard discrete-time linear system in [5–9], as well as feedback shift registers [10]. Moreover, there have been many useful and interesting results in BNs and BCNs, including optimal control [11,12], controllability [13,14], observability [15,16], output regulation [17], solvability [18], attractor transformation [19,20], periodic trajectories [21], synchronization [22–24] etc. In addition, stochastic dynamical networks [25], neural networks [26,27], nonlinear singular systems [28], and complex-valued single neuron model [29] have been further investigated. Also the impulsive disturbances or disturbance decoupling for BCNs have been also studied by Yang et al. [30], Cheng [31] and Gao et al. [32].

Unlike the classical BCNs, probabilistic Boolean control networks (PBCNs) consider the stochastic phenomena, which can not be reflected by general BCNs. Therefore, to study the PBCNs is a very meaningful expansion for classic BCNs. Investigations on PBCNs have been very mature, involving a lot of contents, such as state stabilization [33], weak reachability [34], controllability [35,36], as well as output tracking control problem [37]. In [38], the author has studied the stochastic

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stabilization of n-person random evolutionary Boolean games. Possieri and Teel [39] combined classical deterministic BCNs and PBCNs to investigate global asymptotic stability by constructing Lyapunov functions. In this paper, we consider a more general model named context-sensitive PBCNs (CS-PBCNs), in which the deciding time is randomly selected. It represents that at each time point, there is a probability of whether to switch a new context or remain in the previous context. Generally speaking, the CS-PBCNs are more complicated than those of PBCNs, and accordingly to study control problems of CS-PBCNs will be more difficulty. However, due to its practicality in gene regulatory networks, there are still many significant studies. Pal et al. [40] have treated intervention via external control variables in CS-PBCNs and the results are applied to gene-expression data collected in a study of metastatic melanoma. The authors have used probabilistic model checking to consider optimal control in apoptosis network [41]. In [42], some sufficient conditions for the stability and stabilization of CS-PBCNs have been presented. And for the optimal control in CS-PBCNs, an integer programming approach is used in [43]. There have been some more results on CS-PBCNs, such as robust control of CS-PBCNs [44] and steady-state approximation of CS-PBCNs [45].

In the study of BNs and BCNs, the stabilization or set stabilization are the important issues in the real world which can not be ignored. Li et al. [46] have considered the stabilization problem for BCNs with normal state feedback control. The set stabilization has been investigated in [47] with switched signals and Li et al. [48] investigated the equivalence issue of two kinds of controllers in BCNs. To stabilize systems, the pinning control is used in [49] to control the dynamics of the entire network by selecting key nodes according to some specific properties of nodes. When modeling a biological regulation network, introducing a feedback loop from some specific (output) variables can be viewed either as an attempt to achieve a more complete representation of the network dynamics [50]. Also Li and Wang [51] point out that output feedback controller is one of the possible feedback stabilizers for the disease treatment, which is a meaningful topic in both theoretical developments and practical applications. Meanwhile, the output state feedback controller has been also used to investigate the disturbance decoupling problem for BCNs in [52]. Of course, the output state feedback controllers studied in the above literatures are static output output state feedback controllers.

It is well known that, many experts and scholars have studied a lot of problems about CS-PBCNs, and achieved pretty good results. In the control issue of BCNs, the discussions of set stabilization have been also plentiful. The set stabilization problem of CS-PBCNs with static output feedback control is new and challenging. The main difficulties lie in the following two aspects: i) The dynamics of a CS-PBCNs with static output feedback control is much more complicated than a normal PBCNs, which makes the mathematical derivation tough. ii) In order to make S-stabilizable, finding the largest control invariance set and conditions to ensure stabilization are challenging. To overcome these difficult issues, we should solve the following key problems. First, we need to analyze the logical structure of CS-PBCNs and simplify the transition probability matrix. Then we can find the largest control invariance set for CS-PBCNs. Finally, the static output feedback law matrix K needs to be designed according to the analysis of reachable sets $R_k(S^*)$.

In this paper, we will study the static output feedback set stabilization of CS-PBCNs. And the main contributions of this paper lie in the following three aspects: (i) The algorithm of finding the largest control invariance set S^* is designed. (ii) A necessary and sufficient condition obtained in Theorem 1 for CS-PBCNs to judge whether it is S-stabilizable by an static output feedback controller. (iii) The static output feedback controllers for the S-stabilizable are obtained by Theorem 2.

The rest of this paper is organized as follows. Section 2 provides some preliminaries on STP and gives an algebraic expression of CS-PBCNs. Some necessary and sufficient conditions are obtained for the S-stabilization of CS-PBCNs based on static output feedback control in Section 3. An illustrative example is provided to support the efficiency of the obtained results in Section 4. Finally, a brief conclusion is presented in Section 5.

2. Preliminaries

2.1. Semi-tensor product of matrices

First, some necessary notations are provided in the following.

- $\mathcal{D} := \{1, 0\}$, and $\mathcal{D}^n := \underbrace{\mathcal{D} \times \cdots \times \mathcal{D}}_{n}$.
- Denote $M_{m\times n}$ as the set of all $m \times n$ matrices.
- $\Delta_k := \{\delta_k^i | i = 1, 2, \ldots, k\}$, where δ_k^i is the ith column of identity matrix I_k. For compactness, $\Delta_2 = \Delta$.
- A matrix $A \in M_{m\times n}$ is called a logical matrix if the column set of A, denoted by Col(A), satisfies Col(A)$\subseteq \Delta_m$, and let Col$_i$(A) and Row$_j$(A) be the ith column and the jth row of the matrix A, respectively.
- Denote the set of $n \times m$ logical matrices by $\mathcal{L}_{m\times n}$. If $A \in M_{m\times n}$, it can be expressed as $A = [\delta_m^{i_1}, \delta_m^{i_2}, \ldots, \delta_m^{i_n}]$, or simply by $A = \delta_m[i_1, i_2, \ldots, i_n]$.
- $v = (v_1, v_2, \cdots, v_n,)^T$ is called a probabilistic vector for $\Sigma_{i=1}^n v_i = 1$, and the set of n dimensional probabilistic vectors is denoted by Υ^n.
- Split a matrix M into equal dimension blocks and denote by $Blk_i(M)$ the ith block of matrix M.
- For two Boolean vectors $a = (a_1, a_2 \ldots a_n)^T \in \mathcal{D}^n$, $b = (b_1, b_2 \ldots b_n)^T \in \mathcal{D}^n$, the Boolean addition $\oplus$ is defined as

$$a \oplus b := a \vee b = (a_1 \vee b_1 \ldots a_n \vee b_n)^T \in \mathcal{D}^n.$$

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