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“Follow the leader” learning dynamics on networks

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ABSTRACT

A learning dynamics on network is introduced, characterized by binary and multiple nonlinear interactions among the individuals distributed in the different nodes. A particular topology of the network is considered by introducing a leader node which influences all the other “follower” nodes without being influenced in turn. Numerical simulations are provided, particularly focusing on the effect of the network structure and of the nonlinear interactions on the emerging behaviour of the system. It turns out that the leader node always exhibits an autonomous evolution, while the follower nodes may have a regression when the interactions with the leader node are switched off. There is instead a remarkable change in the final configurations of the follower nodes, even if only one of them is connected to the leader: indeed, due to the microscopic interactions among them, all the follower nodes feel a strong “leader effect”.

1. Introduction

The modeling of intelligent, complex systems is a very active research field which motivated many studies in recent years due to its relevance in life-sciences [1]. In this respect, a very important role is played by collective learning, not only in biological systems but in the field of social sciences as well [2,3]. In order to describe the complexity of the interactions for large systems of living entities, applied mathematicians developed different approaches ranging from kinetic and statistical mechanics models to stochastic methods and Monte Carlo simulations [4–7]. Moreover, in the framework of evolutionary game theory the study of social interactions has been developed through complex networks and structured populations models [8,9]. In the same context, ample attention has been devoted to the role of group leaders and mutual learning, towards a cooperative behaviour [10,11] in population dynamics. On the other hand, in the framework of the so-called kinetic theory for active particles [KTAP theory] [12], several interesting models were obtained in a variety of different fields such as social systems [13], spread of epidemics [14], micro-scale darwinian evolution [15] and collective learning processes [16–18]. Moreover, an interesting survey on the applications of the KTAP theory to economic and social sciences is reported in [19]. More recently, the KTAP theory has been extended to model interactions of groups of individuals localized on networks in order to describe social phenomena such as for example, population migration [20], opinion formation [21–23] or specific problems of political economy [24]. Moreover, a network representation of complex

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learning systems has been presented in [25,26] and applied to describe learning processes that take place in a classroom. It is the aim of this paper to analyze the emerging properties in a population composed by a very large number of interacting entities characterized by a microscopic learning dynamics and distributed over a network connecting a certain number of nodes. A particular topology of the network is considered by introducing a special node having the role of "leader", i.e. influencing all the other nodes of the network, although not being influenced in turn. This is a new aspect with respect to the modeling described in [25,26] which allows to enhance the leader's role in social dynamics.

In the mathematical framework of the KTAP theory, living entities are called active particles: their microscopic state is modeled by a scalar variable called activity which is assumed to be heterogeneously distributed among the particles. The activity represents the ability of each individual to express a given strategy, which, in the case under analysis, is the ability to learn.

In the following we divide the complex system in different populations, mathematically characterized as functional subsystems, localized in the different nodes of the network. Conservation of the size of the population on each node is assumed; the case in which migration phenomena among the nodes are allowed [1], is discussed as a research perspectives. In Section 2 the reference mathematical formalism is introduced and applied to a network of living entities characterized by different abilities to learn. Interactions of the individuals among them induce a learning process which can improve the level of knowledge of the individuals involved, but also in some cases be misleading and reduce it. Section 3 is devoted to the solution of the initial value problem for the mathematical model introduced. Next, a specific learning dynamics on the network is discussed in Section 4. Numerical simulations are provided in Section 5 especially focusing on the effect of the network structure on the emerging behaviours of the system. Finally, research perspectives are outlined in Section 6.

2. Mathematical framework

In the following we introduce the essential aspects of the reference mathematical formulation and specialize them to the case of living entities interacting on a network. Moreover, we characterize the properties of the network and of the nonlinear interactions involved in the learning dynamics of the complex system under study.

2.1. Mathematical representation

The hallmarks of the KTAP theory are the following:

- The overall system is subdivided into functional subsystems constituted by entities, called active particles, whose individual state is called activity;
- the state of each functional subsystem is defined by a suitable, time dependent, probability distribution over the activity variable;
- interactions are modeled by games, more precisely stochastic games [17], where the state of the interacting particles and the output of the interactions are known in probability;
- the evolution of the above mentioned probability distributions is obtained by a balance of particles within elementary volumes of the space of the microscopic states,where the dynamics of the inflow and the outflow of particles is determined by the interactions at the microscopic scale.

In particular, in the complex system under analysis, each of the active particles expresses a learning strategy at any given instant of time. The active particle's strategy is represented by a scalar variable $u \in D_u \subset \mathbb{R}$, called activity, which is heterogeneously distributed among the particles. A discrete normalized domain is assumed for the activity variable on which it takes m discrete states, i.e. $D_u = \{u_1 = 0, \dots, u_r = \frac{r-1}{m-1}, \dots, u_m = 1\}$, ranging from the worst learning activity $u_1 = 0$ to the best one $u_m = 1$.

We introduce n subsystems characterized by the same activity variable u, although it will be expressed on each subsystem in different ways by means of different parameters and/or different initial conditions. On each subsystem a time evolving discrete probability on the active variable is introduced

$$f_r^i = f^i(u_r, t) \,:\, D_u \times [0, T_{max}] \to [0, 1], \quad i = 1, \dots, n, \quad r = 1, \dots, m, \tag{1}$$

which has been normalized with respect to the constant size of the population; T_{max} is the observation time. The following probability property holds:

$$\sum_{r=1}^{m} f_r^i(t) = 1, \quad \forall\, i = 1, \dots, n \quad \forall t \geq 0$$

on each subsystem in each instant of time. The notation $\mathbf{f}^i = (f_1^i, \dots, f_m^i)$ will be used in the following for the probability on the ith node; on each node we introduce the first moment of the distribution

$$\mathbb{M}^{(i)}[\mathbf{f}(t)] = \sum_{r=1}^{m} u_r \mathbf{f}_r^i(t), \quad \forall i = 1, \dots, n.$$

In the present model we consider two kinds of interactions: