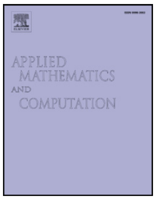
Applied Mathematics and Computation 332 (2018) 351–362

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc



Conduction and convection heat transfer in a dense granular suspension

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ARTICLE INFO

Keywords:
Non-Fourier heat conduction
Granular materials
Non-Newtonian fluid
Suspension

ABSTRACT

Flow and heat transfer in a suspension down an inclined plane is studied; non-linear constitutive relations for the stress tensor and the heat flux vector are used. The (material) coefficients appearing in these constitutive relations are assumed to be functions of the volume fraction. Different thermal boundary conditions including radiation boundary condition at the free surface are used and a parametric study is performed to study the impact of the dimensionless numbers on the flow and heat transfer. The dimensionless forms of the governing equations are solved numerically, and velocity, volume fraction and temperature fields are obtained.

1. Introduction

Flow of an assembly of (dense) solid particles, their interactions with each other and their impact on environment, such as corrosion, wear, erosion, deposition, etc., is an important area of research in chemical processes such as solids bulk handling. To study these problems, one can take the statistical approach, i.e., use certain averaging procedures to obtain constitutive relations. Numerical simulations can provide some insight into particles motion. It is also possible to use continuum mechanics where constitutive relations are derived based on some physically observed phenomena. In many applications, it is necessary to use the multi-component (or multiphase) approach. In some situations, one can neglect the effect of the interstitial fluid and as a result one can consider the granular materials as a single phase, or a single component continuum, otherwise a multiphase modeling approach should be used. The review article by Hutter and Rajagopal [1], and books by Nedderman [2] and Mehta [3] discuss many of the relevant issues in granular materials.

Flowing suspensions composed of solid particles occur in many industrial applications such as storage and transport of coal and grains. These suspensions, in general, have different rheological properties, and they are composed of particles with different shapes and sizes. The amount of moisture content, the environment temperature, or the presence of electrostatic forces, etc. are some of the parameters which will affect the flow and heat transfer in these complex non-linear [4–7]. Nedderman [2] indicates that roughly half of the products and more than three quarters of the raw materials used in chemical industries are in granular form. In many of these processes, it is necessary to know the velocity and the temperature fields, and therefore accurate and reliable knowledge of the (effective) thermal conductivity and viscosity of the suspensions are needed. Although heat transfer processes are important in some chemical processes, the problem of thermal convection has only recently received attention. Sometimes, these materials are also heated prior to processing, or cooled after processing

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https://doi.org/10.1016/j.amc.2018.03.056
0096-3003/Published by Elsevier Inc.

(see Patton et al. [8]). These dense phase flows occur in many industrial equipment designed to heat, cool, or dry granular materials (see Uhl and Root [9]). In recent years, heat transfer in granular materials has been studied both experimentally and theoretically, especially heat transfer from a flat plate to various particulate materials and heat transfer to granular materials flowing along an inclined chute at higher velocities (see Sullivan and Sabersky [10]). Some experimental results indicate that the higher velocities can cause a decrease in the density of the material and this decrease in density can cause a reduction in heat transfer (see Spelt et al. [11]). Gudhe et al. [12] considered the flow of granular materials down a heated inclined plane; they included the effect of viscous dissipation in their analysis. In general, a complete thermodynamic analysis of the constitutive equations is missing. To understand the rheological and thermal performance of these materials, it is crucial to have reliable formulations for the stress tensor and the heat flux vector. In most applications, the Fourier's law of heat conduction is used, and an effective thermal conductivity is proposed; however, it is known that for many complex materials the heat flux vector can depend not only on the temperature gradient but also on the shear rate, and other parameters (see Massoudi [13,14]).

In this paper, we use a simplified version of the model proposed by Massoudi [13,14] and study the one dimensional fully developed flow of a dense granular suspension down an inclined plane. The equations are made dimensionless; we perform a parametric study to see the effects of the additional parameters on the heat flux vector. The equations are coupled non-linear second order ordinary differential equations which are solved numerically, and the results are shown for the temperature, volume fraction and velocity profiles.

2. The governing equations

In the absence of chemical and electromagnetic effects, the governing equations of motion are the conservation equations for mass, linear and angular momentum and energy equations [15]

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \boldsymbol{v}) = 0 \tag{1}$$

$$\rho \frac{d\boldsymbol{v}}{dt} = \mathrm{div}\ \boldsymbol{T} + \rho \boldsymbol{b} \tag{2}$$

$$\boldsymbol{T} = \boldsymbol{T}^{\boldsymbol{T}} \tag{3}$$

$$\rho \frac{d\varepsilon}{dt} = \boldsymbol{T} : \boldsymbol{L} - \mathrm{div}\ \boldsymbol{q} + \rho r \tag{4}$$

where

$$\rho = \rho_s \phi \ (0 \le \phi < \phi_{\max} < 1) \tag{5}$$

where, ρ is the density, t is time, $\boldsymbol{v}$ is the velocity vector, $\frac{d}{dt}$ is the material derivative, $\boldsymbol{T}$ is the Cauchy stress tensor, $\boldsymbol{b}$ is the body force, ε is the internal energy, $\boldsymbol{q}$ is the heat flux, $\boldsymbol{L}$ is velocity gradient tensor, r is the radiant heating, ρ_s is the (pure) density of the particle, and ϕ is the volume fraction. Physically, there is a packing limit where the volume fraction is constrained by an upper bound $\phi_{\max}$ which is less than 1. The specific value of $\phi_{\max}$ depends on the shape and size distribution of the particles.

In addition to these equations, the second law of thermodynamics should also be used (see Liu [16]):

$$\rho \dot{\eta} + \mathrm{div}\ \boldsymbol{\varphi} - \rho s \ge 0 \tag{6}$$

where $\eta(\boldsymbol{x}, t)$ is the specific entropy density, $\boldsymbol{\varphi}(\boldsymbol{x}, t)$ is the entropy flux, s is the entropy supply density due to external sources, and the superposed dot denotes the material time derivative. If we assume $\boldsymbol{\varphi} = \frac{1}{\theta}\boldsymbol{q}$ and $s = \frac{1}{\theta} r$, where θ is the absolute temperature, then the above inequality Eq. (6) reduces to the more familiar Clausius–Duhem inequality

$$\rho \dot{\eta} + \mathrm{div} \frac{\boldsymbol{q}}{\theta} - \rho \frac{r}{\theta} \ge 0 \tag{7}$$

In many studies, this inequality has been used to obtain restrictions on the constitutive relations (see Coleman and Noll [17], Dunn and Fosdick [18], Fosdick and Rajagopal [19]). We can see from Eq. (1) to (7), that constitutive equations for $\boldsymbol{T}$, $\boldsymbol{q}$, ε and r are needed before any problem can be solved. In the next section, we will provide a brief discussion of the constitutive relations for the stress tensor and the heat flux vector.

3. The constitutive equations

Due to the complex and non-linear nature of granular materials where they show fluid-like and solid-like behavior depending on the condition, there is no single constitutive relation accepted by the community which can capture features such as yield stress, normal stress effects, viscoelastic response, etc. We will not discuss these issues here and refer the reader to review articles by Savage [20], Hutter and Rajagopal [1], Massoudi [21] and the books by Rao and Nott [22] and