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Stability analysis and control synthesis for positive semi-Markov jump systems with time-varying delay

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ABSTRACT

This paper deals with stability analysis and control synthesis for positive semi-Markov jump systems (S-MJSs) with time-varying delay, in which the stochastic semi-Markov process related to nonexponential distribution is considered. The main motivation for this paper is that the positive condition sometimes needs to be considered in S-MJSs and the controller design methods in the existing have some conservation. To deal with these problems, the weak infinitesimal operator is firstly derived from the point of view of probability distribution under the constraint of positive condition. Then, some sufficient conditions for stochastic stability of positive S-MJSs are established by implying the linear Lyapunov–Krasovskii functional depending on the bound of time-varying delay. Furthermore, an improved stabilizing controller is designed via decomposing the controller gain matrix such that the resulting closed-loop system is positive and stochastically stable in standard linear programming. The advantages of the new framework lie in the following facts: (1) the weak infinitesimal operator is derived for S-MJSs with time-varying delay under the constraint of positive condition and (2) the less conservative stabilizing controller is designed to achieve the desired control performance. Finally, three examples, one of which is the virus mutation treatment model, are given to demonstrate the validity of the main results.

1. Introduction

Positive systems [1,2] are a special class of dynamic systems, in which the state variables and output signals remain nonnegative whenever both the initial condition and the input signal are nonnegative. Positive systems have extensive applications in communication systems [3], virus mutation systems [4], system ecology [5,6], market economics, chemical industry, and environmental science, and have attracted ever-increasing research interests from different fields, so as to promote the development of positive system theory [7–12]. To mention a few, a necessary and sufficient condition for the existence of common linear copositive Lyapunov function was proposed for switched systems with two constituent linear time-invariant systems [7]. The synthesis of state-feedback controllers for positive linear systems was solved in terms of

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linear programming problem, including the requirement of positiveness of the controller and its extension to uncertain plants [9]. Under simplifying assumptions, viral mutation treatment dynamics could be viewed as a positive switched linear system [11]. Using linear co-positive Lyapunov functions, results for the synthesis of stabilizing, guaranteed performance and optimal control laws for switched linear systems were presented. By the use of simple linear performance index, linear Lyapunov function, cone invariant set with linear form and linear computation tool, a new model predictive control framework was proposed for positive systems subject to input/state constraints and interval/polytopic uncertainty [12].

On the other hand, time delay exists widely in practical systems, which has been regard as an important factor to make performance worse and system out of control [13–15]. Time delay increases the difficulty of theoretical analysis and engineering application, and also brings some challenges to analysis and synthesis of dynamics. Over the past decades, considerable attention has been devoted to positive systems with time delay (see e.g., [16–19]). For continuous-time delayed linear systems [16], some necessary and sufficient conditions were presented for the existence of controllers with bounded control satisfying the positivity constraint. By virtue of the positivity and linearity of the system, an explicit expression of the $\mathcal{L}_\infty$-gain of positive systems with distributed delays was given in terms of system matrices [17]. By using a copositive Lyapunov–Krasovskii functional and the average impulsive interval method, a sufficient criterion of global exponential stability for delayed impulsive positive systems was established in terms of linear programming problems [18]. For switched nonlinear positive systems with time-varying delay [19], by using the nonlinear Lyapunov–Krasovskii functional, sufficient conditions for absolute exponential $\mathcal{L}_1$ stability and a state-feedback control law were established in terms of linear programming.

Furthermore, as a special kind of hybrid systems [20–26], Markov jump systems (MJSs) [27–38] have shown some advantages of describing various physical systems, such as economics systems, manufacturing systems, power systems, network-based control systems, and other aspects. Recently, many results about positive MJSs have been reported; for details, see [39–48]. To mention a few, for positive Markov jump linear systems [39], the equivalent relations between exponential mean stability and 1-moment stability were built and various sufficient conditions for exponential almost-sure stability were worked out with different levels of conservatism. For positive systems with Markovian jump parameters [40], the problems of stochastic stability and mode-dependent state-feedback controller design in both continuous-time and discrete-time contexts were addressed. By constructing a linear stochastic Lyapunov function and introducing an equivalent deterministic positive discrete-time linear system, necessary and sufficient conditions for stochastic stability and ℓ_1-gain performance of the positive discrete-time MJLS were derived and the existence of the desired positive ℓ_1-gain filter was provided by solving a standard linear programming problem [41]. By using a copositive Lyapunov function approach, a computable sufficient condition for positive Markov jump linear systems with switching transition probability was proposed in the framework of dwell time to guarantee the mean stability [42]. Necessary and sufficient conditions of mean stability and stabilization were established for both continuous-time and discrete-time positive Markov jump systems in terms of standard linear programming [43]. For Takagi–Sugeno fuzzy positive Markovian jump systems [48], by constructing a linear copositive Lyapunov function, a positive $\mathcal{L}_1$ state-bounding observer was designed in linear programming.

As we know, the sojourn time of the transition rate in MJSs follows unique memoryless exponential distribution, which means that the transition rate becomes time-invariant independent of the sojourn time. Compared with MJSs, the ST in S-MJSs obeys a more general nonexponential distribution including Weibull distribution, phase type distribution, Gaussian distribution, and so on, which leads to the time-varying characteristic of the transition rate matrix and brings both challenges and chances to analysis and synthesis of dynamics. Very recently, S-MJSs have many applications including hospital planning, multiple-bus systems, and credit risk systems. To date, it is noted that analysis and synthesis of S-MJSs have attracted a wide range of research interests (see e.g., [49–61]). For semi-Markov jump linear systems with norm-bounded uncertainties [50], the robust stochastic stability condition and the robust control design problem were addressed. For a class of continuous-time semi-Markovian jump systems [51], the possibility to explore more refined probability models on the sojourn time was offered by considering the mode transition-dependent sojourn-time distributions instead of mode-dependent ones. By using a supplementary variable technique and a plant transformation, a finite phase-type semi-Markov process was transformed into a finite Markov chain and a sliding mode controller was synthesized to guarantee the associated Markovian jump systems satisfying the reaching condition [52]. Robust passivity-based sliding mode controller for uncertain singular systems with semi-Markov switching and actuator failures was designed to ensure the closed-loop system to be stochastically admissible and robustly passive [54]. For a class of stochastic neutral-type neural networks with both semi-Markovian jump parameters and mixed time delays [57], a Luenberger-type observer was designed to guarantee the filter error dynamics mean-square exponentially stable with an expected decay rate and an attenuation level. For a class of semi-Markovian jump systems with external disturbance and sensor fault [59], an observer-based sliding mode control scheme was proposed to stabilize the fault closed-loop system and the reachability of the proposed sliding mode surface could be guaranteed.

In this paper, the problems of stability analysis and control synthesis for positive S-MJSs with time-varying delay are considered. It is necessary to point out the differences between the present work and some existing relative works [6,9,39–61]. First, for controller design method, the iterative optimization method has been introduced in [6] to compute the state feedback controller gain matrices, which increases the calculation complexity especially in the presence of high-order system. The parameter selection method has been presented in [40,44,45,47] with the rank of controller gain matrix confined to one. The dual system design method [9,43] could only be applied to the nominal system and could not be extended to more complex systems with nonlinearity and parameter uncertainties. Second, in the literatures [39–48], the sojourn time