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Number of proper paths in edge-colored hypercubes

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A R T I C L E I N F O

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A B S T R A C T

Given an integer $1 \leq j < n$, define the (j)-coloring of a n-dimensional hypercube H_n to be the 2-coloring of the edges of H_n in which all edges in dimension i, $1 \leq i \leq j$, have color 1 and all other edges have color 2. Cheng et al. (2017) determined the number of distinct shortest properly colored paths between a pair of vertices for the (1)-colored hypercubes. It is natural to consider the number for (j)-coloring, $j \geq 2$. In this note, we determine the number of different shortest proper paths in (j)-colored hypercubes for arbitrary j. Moreover, we obtain a more general result.

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1. Introduction

In this work, all graph colorings are colorings of the edges. A coloring of a graph (or subgraph) is called *proper* if no two adjacent edges in the graph (respectively, subgraph) have the same color. A colored graph is called *properly connected* if there is a properly colored path between every pair of vertices [1].

Given a properly connected coloring of a graph, Coll et al. [3] initiated the study of the proper distance between pairs of vertices. The *proper distance* between two vertices u and v, denoted by $pd(u, v)$, is the minimum length (number of edges) of a properly colored path between u and v. Although the proper connection number has been studied and applications to computer science and networking have been described, precious little attention has been paid to the proper distance which is, in some sense, a measure of the efficiency of the properly connected network [2].

Numerous works related to proper connectivity and the proper connection number have been studied in recent years, see the dynamic survey [6] for more details. The study of proper connection in graphs is motivated in part by applications in networking and network security. In addition to security, of course it is critical that message transmission in a network be fast [2]. So we consider the number of different shortest proper colored paths between any pair of vertices in (j)-colored hypercubes.

An n-dimensional hypercube H_n (with $n \geq 2$) is an undirected graph $H_n = (V, E)$ with $|V| = 2^n$ and $|E| = n2^{n-1}$. Every vertex can be represented by an n-bit binary string. There is an edge between two vertices whenever their binary string representation differs in only one bit position. The i-dimensional edges of hypercubes is the set of edges whose two ends have the different ith bit position. Given $1 \leq j < n$, define the (j)-coloring of H_n to be the 2-coloring of the edges of H_n in which all edges in dimension i, where $1 \leq i \leq j$, have color 1 and all other edges have color 2. For example, the (1)-coloring of H_n has a perfect matching in color 1 and all other edges in color 2.

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For any two vertices u and v of H_n, let $o(u, v)$ denote the number of positions i for $1 \le i \le j$ in which u and v differ. Let $t(u, v)$ denote the number of positions i for $j < i \le n$ in which u and v differ. Let $\gamma(u, v)$ be the indicator variable which takes the value 1 if $o(u, v) + t(u, v)$ is odd and 0 if $o(u, v) + t(u, v)$ is even.

Cheng et al. proved the proper distance between any two vertices of a (j)-colored hypercube.

Theorem 1.1 [2]**.** *Let $n \ge 2$, $1 \le j < n$, and H be the (j)-coloring of H_n. Then H is properly connected. Furthermore, let u and v be any pair of vertices in H. Then*

$$pd(u, v) = 2max(o(u, v), t(u, v)) - \gamma(u, v).$$

For the number of the shortest properly colored paths in edge-colored hypercubes, there are no general results, Cheng et al. determined the special case $j = 1$ in (j)-colored hypercubes [2]. Let $pp(u, v)$ denote the number of distinct shortest properly colored paths in H from u to v and $a(m)$ denote the number of paths between u and v of length $2m$, where $m = \lceil \frac{pd(u,v)}{2} \rceil$. In addition, if we consider the first j bits, let $a(m) = a_o(m)$; Otherwise, let $a(m) = a_t(m)$.)

Theorem 1.2 [2]**.** *Let $n \ge 2$ and let H be the (1)-coloring of H_n. Then H is properly connected. Furthermore, let u and v be any pair of distinct vertices whose strings differ in $t = t(u, v)$ positions other than the first. If $t = 0$, then u and v are adjacent so $pd(u, v) = 1$ and $pp(u, v) = 1$. Otherwise, for $t \ge 1$, we have $pd(u, v) = 2t - \gamma(u, v)$ and $pp_H(u, v) = (2 - \gamma(u, v))(t!)$.*

In next section, we determine the case for $j > 1$. Moreover, a more general result will be discussed. For the structural properties we refer to [4,5,7–9].

2. Number of proper paths between vertices

In this section, we shall count the number of different shortest proper paths between any two vertices in a (j)-colored hypercube. Cheng et al. induced a method to find properly colored paths between two given vertices u to v. We will follow the method in our proof to find shortest properly colored paths between two given vertices. We give an example as follows:

Example 2.1 [2]**.** If $n = 7$ and $j = 4$ and we consider $u = (0, 1, 0, 1, 0, 0, 0)$, and $v = (0, 0, 1, 1, 1, 1, 1)$, then some examples of proper colored paths from u to v created by the algorithm above would be as follows:

$$\begin{aligned} u = {} & (0, 1, 0, 1, 0, 0, 0) \rightarrow \\ & (0, 0, 0, 1, 0, 0, 0) \rightarrow \\ & (0, 0, 0, 1, 1, 0, 0) \rightarrow \\ u' = {} & (0, 0, 1, 1, 1, 0, 0) \rightarrow \\ & (0, 0, 1, 1, 1, 1, 0) \rightarrow \\ & (1, 0, 1, 1, 1, 1, 0) \rightarrow \\ & (1, 0, 1, 1, 1, 1, 1) \rightarrow \\ v = {} & (0, 0, 1, 1, 1, 1, 1) \end{aligned}$$

This is an arbitrary proper colored path.

$$\begin{aligned} u = {} & (0, 1, 0, 1, 0, 0, 0) \rightarrow \\ & (0, 1, 0, 1, 1, 0, 0) \rightarrow \\ & (0, 0, 0, 1, 1, 0, 0) \rightarrow \\ & (0, 0, 0, 1, 1, 1, 0) \rightarrow \\ u' = {} & (0, 0, 1, 1, 1, 1, 0) \rightarrow \\ v = {} & (0, 0, 1, 1, 1, 1, 1) \end{aligned}$$

This is a shortest proper colored path.

In fact, in the hypercube, any path from u to v consists flipping the bits of u until arrive at v [2]. A properly colored path from u to v consists of flipping the bits of u until we arrive at v with the restriction that the bits we flip must alternate between the first j bits and the last $n - j$ bits. To construct such a properly colored path, flip the bits of u that differ with the respective bits of v, making sure to flip the bits in an order that alternates between the first j bits and the last $n - j$ bits, until you arrive at a vertex u' such that either $o(u', v) = 0$ or $t(u', v) = 0$.

If $o(u', v) = 0$, we alternate flipping the first bit with flipping the remaining bits in the last $n - j$ positions that differ with the respective bits of v. Otherwise if $t(u', v) = 0$, then we alternate flipping the last bit with flipping the remaining bits in the first j positions that differ with the respective bits of v. Since there are a finite number of positions in which u and v differ and the process will not repeat vertices, this process will eventually terminate, resulting in a properly colored path from u to v [2].

Certainly the path contains at least $o(u, v)$ bits flipped from the first j bits and at least $t(u, v)$ bits flipped from the last $n - j$ bits. In every properly colored path, the flipping must alternate between the first j bits and the last $n - j$ bits, there

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