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A well-balanced element-free Galerkin method for the nonlinear shallow water equations

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ABSTRACT

In this paper, we consider the nonlinear shallow water equations over variable bottom topography in one dimension and propose a well-balanced element-free Galerkin method for solving this system. The proposed scheme has the features of being high-order accurate for general solutions and exactly preserving the still-water stationary solution. The main ingredient to achieve the well-balanced property is to use a special decomposition to the source term and discretize the source term as the flux term. Numerical tests are presented to illustrate the accuracy and validity of the proposed scheme.

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1. Introduction

The nonlinear shallow water equations (NSWEs) have been widely applied to problems in ocean and hydraulic engineering. In this paper, we consider the NSWEs in one-dimensional (1D) space, which read

$$\begin{cases} h_t + (hu)_x = 0, \\ (hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = -ghb_x, \end{cases} \tag{1}$$

where h denotes the water depth, u is the vertically averaged horizontal velocity, b represents the bottom topography and g is the gravitational constant. The subscripts t and x denote the partial derivatives with respect to time t and the coordinate x, respectively.

The NSWEs have still-water stationary solutions

$$h + b = \text{constant}, \qquad hu = 0, \tag{2}$$

for which the flux gradients are non-zero and exactly balanced by the source terms. Such stationary solutions as well as their perturbations, are difficult to capture numerically. Many traditional numerical methods usually fail to preserve such solutions, and thus numerical spurious oscillation occurs in the simulation. To overcome this issue, many researchers have proposed a number of well-balanced schemes in the past few decades. LeVeque [1] developed a quasi-steady wave-propagation algorithm by introducing a Riemann problem in the center of each grid cell whose flux difference exactly cancels the source term. Audusse et al. [2] designed a well-balanced finite volume method for the shallow water equations. Xing and Shu [3] proposed well-balanced finite volume WENO schemes and discontinuous Galerkin Methods for the shallow water equations. Li et al. [4] developed well-balanced central discontinuous Galerkin methods for the shallow water equations. Other well-balanced methods for the shallow water equations include [5–13] and the references therein.

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Another difficulty in the simulation of shallow water waves is the appearance of dry areas or near-dry areas, where no water or little water is present. If no special attention is paid, negative water depth may be produced due to numerical errors, possibly leading to computation breakdown. Recently, many works have been developed to maintain the non-negativity of the water depth. Xing et al. [14] designed a positivity-preserving discontinuous Galerkin Methods for the shallow water equations by using a positivity-preserving limiter. Noelle et al. [15] proposed a positivity-preserving finite volume method for the shallow water equations based on a continuous, piecewise linear discretization of the bottom topography. Vater et al. [16] developed a new algorithm to model inundation events with piecewise linear Runge–Kutta discontinuous Galerkin approximations applied to the shallow water equations and discussed the stability of the algorithm in case of rapid transition of the wet/dry interface. Li et al. [17] proposed positivity preserving central discontinuous Galerkin methods for the shallow water equations with a similar limiter in [14]. We also refer the readers to [18–23] and the references therein for other positivity-preserving schemes.

In this paper, we shall focus on the first issue. We propose a high-order well-balanced scheme based on the improved element-free Galerkin method for the NSWEs (1). The main ingredient to achieve the well-balancing property is to use a special decomposition to the source term similar to that proposed in [3,4,24] and discretize the source term as the flux term. The proposed scheme has the features of being high-order accurate for general solutions and exactly preserving the still-water stationary solution.

The element-free Galerkin (EFG) method, originally introduced for elasticity and heat conduction problems [25], is a class of meshless methods based on the moving least-squares (MLS) approximation. The EFG method has been applied to various problems [26–29]. In Ref. [29], the authors applied the EFG method to solve the shallow water equations, however they did not discuss the well-balancing property and they only showed the numerical results on flat bottom. Since the MLS approximation can sometimes form an ill-conditioned equation system, the improved MLS approximation, which uses orthogonal function system with a weight function as the basis function, is proposed by Cheng and Peng [30] and Liew et al. [31]. By replacing the MLS approximation with the improved MLS approximation in the element-free Galerkin method, an improved EFG method has been developed [32–35]. In the improved EFG method, fewer nodes are needed in the entire problem domain than those are needed in the EFG method. Hence, the improved EFG method has a greater computational efficiency.

This paper is organized as follows. In Section 2, we present a well-balanced element-free Galerkin method for the NSWEs (1), we also prove the well-balancing property of the proposed method. Then, we show several numerical tests to illustrate the accuracy and reliability of the proposed method in Section 3. Finally, concluding remarks are given in Section 4.

2. Numerical schemes

In this section, we develop a well-balanced numerical method for the solution of 1D NSWEs. For ease of presentation, we rewrite (1) as

$$\begin{cases} h_t + q_x = 0, \\ q_t + f(h,q)_x = s(h;b), \end{cases} \tag{3}$$

where $q = hu$, $f(h,q) = \frac{q^2}{h} + \frac{1}{2}gh^2$ and $s(h;b) = -ghb_x$.

Let $\Omega = (a_0, a_1)$ denote the computational domain and $\{x_j\}_{j=1}^N$ be an arbitrarily chosen set of N nodes $x_j \in \bar{\Omega}$. We use $\Delta x = \max_{1\le i\le N-1} |x_i - x_{i+1}|$ to denote the nodal spacing.

Besides, the numerical solution, which are assumed to be available at $t = t_n$, denoted by h^n, q^n, and we want to find the solutions at $t = t_{n+1} = t_n + \Delta t_n$, with Δt_n being the time step.

2.1. The improved MLS approximation

In this section, we briefly introduce the improved MLS approximation, more details can be found in [30,31]. In the improved MLS approximation, a given function $v(x)$ defined on Ω can be approximated by

$$v(x) \simeq v_h(x) = \sum_{i=1}^{m} p_i(x) a_i(x) = \mathbf{p}^T \mathbf{a} \tag{4}$$

where $\mathbf{p}^T = (p_1(x), p_2(x), \ldots, p_m(x))$ is a vector of basis functions which satisfies

$$(p_k, p_i) = \sum_{j=1}^{N} w(x - x_j) p_k(x_j) p_i(x_j) = \begin{cases} 0, & k \neq i, \\ \alpha_k(x), & k = i, \end{cases} \tag{5}$$

and $w(x - x_j)$ is a weight function with compact support, $\mathbf{a} = (a_1(x), a_2(x), \ldots, a_m(x))^\top$ is the vector of unknown parameters, which is determined at any point x, by minimizing a weighted L_2 norm

$$J(x) = \sum_{j=1}^{N} w(x - x_j)\big(v_h(x_j) - v_j\big). \tag{6}$$

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