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The g-good neighbor conditional diagnosability of twisted hypercubes under the PMC and MM* model

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ABSTRACT

Connectivity and diagnosability are important parameters in measuring the fault tolerance and reliability of interconnection networks. The g-good-neighbor conditional faulty set is a special faulty set that every fault-free vertex should have at least g fault-free neighbors. The R^g-vertex-connectivity of a connected graph G is the minimum cardinality of a g-good-neighbor conditional faulty set $X \subseteq V(G)$ such that $G - X$ is disconnected. The g-good-neighbor conditional diagnosability is a metric that can give the maximum cardinality of g-good-neighbor conditional faulty set that the system is guaranteed to identify. The twisted hypercube is a new variant of hypercubes with asymptotically optimal diameter introduced by X.D. Zhu. In this paper, we first determine the R^g-vertex-connectivity of twisted hypercubes, then establish the g-good neighbor conditional diagnosability of twisted hypercubes under the PMC model and MM* model, respectively.

1. Introduction

A multiprocessor system comprises two or more processors, and various processors exchange information via links between them. With the rapid development of science and technology, the number of processors is increasing. As the size of multiprocessor systems increase, processor failure is inevitable. Therefore, the topic of reliability and fault-tolerance of multiprocessor systems has become an active area of intensive research.

To evaluate the reliability of multiprocessor systems, fault diagnosability has become an important metric. There are several different diagnosis models being proposed to determine the diagnosability of a system. In this paper, we use the PMC model introduced by Preparata et al. [18] and MM* model suggested by Sengupta and Dahbura [19]. In the PMC model, every processor can test the processor that is adjacent to it. In the MM* model, every processor must test two processors if the processor is adjacent to the latter two processors. Many researchers have applied the PMC model and MM* model to identify faults in various topologies, see [1–3,9,16].

The diagnosability for multiprocessor systems assumes that all the neighbors of any processor may fail simultaneously. Therefore, for a system, its diagnosability is restricted by its minimum degree. However, the probability that this event occurs is very small in large-scale multiprocessor systems. To obtain a more practical diagnosability, Lai et al. [11] introduced

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conditional diagnosability under the assumption that all the neighbors of any processor in a multiprocessor system cannot be faulty at the same time. The conditional diagnosability of interconnection networks has been extensively investigated, see [2,4,7–10,25,30].

Recently, Peng et al. [17] proposed a new measure for fault diagnosis of the system, namely, g-good-neighbor conditional diagnosability, which requires that every fault-free node contains at least g fault-free neighbors. Compared with diagnosability, the g-good-neighbor conditional diagnosability improves accuracy in measuring the reliability of interconnection networks in heterogeneous environments. Meanwhile, Peng et al. [17] studied the g-good-neighbor conditional diagnosability of the n-dimensional hypercube Q_n under the PMC model. Wang and Han [22] determined the g-good-neighbor diagnosability of the n-dimensional hypercube Q_n under the MM* model. Since then, numerous studies have been investigated under the PMC model and MM* model, see [5,13,14,21,23,24,27,28].

In this paper, we consider the g-good-neighbor conditional diagnosability of the n-dimensional twisted hypercube H_n, which is a new variant of hypercubes with asymptotically optimal diameter introduced by Zhu [29], under the PMC model and MM* model, respectively. Our main results are listed below.

Theorem 1.1. *Let n be an integer with $n \geq 4$. Then the g-good-neighbor conditional diagnosability of H_n under the PMC model is*

$$t_g(H_n) = \begin{cases} 2^g(n-g+1)-1 & \text{if } 0 \leq g \leq n-3, \\ 2^{n-1}-1 & \text{if } g-2 \leq g \leq n-1. \end{cases}$$

Theorem 1.2. *Let n be an integer with $n \geq 5$. Then the g-good-neighbor conditional diagnosability of H_n under the MM* model is*

$$t_g(H_n) = \begin{cases} 2^g(n-g+1)-1 & \text{if } 0 \leq g \leq n-3, \\ 2^{n-1}-1 & \text{if } g-2 \leq g \leq n-1. \end{cases}$$

The rest of this paper is organized as follows: Section 2 provides preliminaries for our notations, diagnosing a system and the twisted hypercubes. In Section 3, we determine the R^g-vertex-connectivity of twisted hypercubes. In Sections 4, we establish the g-good-neighbor conditional diagnosability of twisted hypercubes under the PMC model and MM* model, respectively. Our conclusions are given in Sections 5.

2. Preliminaries

2.1. Notations

Let $G = (V(G), E(G))$ be a simple and finite graph. The *neighborhood* $N_G(v)$ of a vertex v is the set of vertices adjacent to v and the *closed neighborhood* of v is $N_G[v] = N_G(v) \cup \{v\}$. The *degree* $d_G(v)$ of v is $|N_G(v)|$. The *minimum degree* of G is denoted by $\delta(G)$. If $d_G(v) = k$ for any $v \in V(G)$, then G is called a *k-regular* graph. For $S \subseteq V(G)$, $G[S]$ denotes the subgraph induced by S. The *neighborhood set* of S is defined as $N_G(S) = (\cup_{v\in S} N_G(v)) - S$, and the *closed neighborhood set* of S is defined as $N_G[S] = N_G(S) \cup S$. We will use $G - S$ to denote the subgraph $G[V(G) - S]$. For any $v \in V(G)$, $N_S(v)$ denotes the neighborhood of v in S. For two disjoint subsets S, T of $V(G)$, let $E_G(S, T) = \{uv \in E(G) \mid u \in S,\ v \in T\}$. The *symmetric difference* of $F_1 \subseteq V(G)$ and $F_2 \subseteq V(G)$ is defined as the set $F_1 \Delta F_2 = (F_1 - F_2) \cup (F_2 - F_1)$. Let K_n be a complete graph of order n.

The minimum cardinality of a vertex set $S \subseteq V(G)$ such that $G - S$ is disconnected or has only one vertex, denoted by $\kappa(G)$, is the *connectivity* of G. A subset $F \subseteq V(G)$ is called a *R^g-vertex-set* of G if $\delta(G - F) \geq g$. A *R^g-vertex-cut* of a connected graph G is a R^g-vertex-set F such that $G - F$ is disconnected. The *R^g-vertex-connectivity* of G, denoted by $\kappa^g(G)$, is the cardinality of a minimum R^g-vertex-cut of G. Note that $\kappa^0(G) = \kappa(G)$.

2.2. The PMC model and MM* model for diagnosis

A multiprocessor system is typically represented by an undirected simple graph $G = (V, E)$, where $V(G)$ stands for the processors and $E(G)$ represents the links between two processors.

Preparata et al. [18] proposed the PMC model, which performs diagnosis by testing the neighboring processor via the links between them. Under the PMC model, tests can be performed between any two adjacent vertices u and v. We use the ordered pair (u, v) to denote a test that u diagnoses v. The result of a test (u, v) is reliable if and only if u is fault-free. In this condition, the result is 0 if v is fault-free, and is 1 otherwise.

A test assignment for a system G is a collection of tests, which can be represented a directed graph $T = (V(G), L)$, where $V(G)$ is the vertex set of G and $L = \{(u, v) \mid u, v \in V(G) \text{ and } uv \in E(G)\}$. The collection of all test results from the test scheme T is termed as a syndrome $\sigma\colon L \to \{0, 1\}$. Let $T = (V(G), L)$ be a test assignment, and F a subset of $V(G)$. For any given syndrome σ resulting from T, F is said to be consistent with σ if the syndrome σ can be produced when all vertices in F are faulty and all vertices in $V(G) - F$ are fault-free. That is, if u is fault-free, then $\sigma((u, v)) = 0$ if v is fault-free, and $\sigma((u, v)) = 1$ if v is faulty; if u is faulty, then $\sigma((u, v))$ can be either 0 or 1 no matter v is faulty or not. Therefore, a faulty set F may be consistent with different syndromes. We use $\sigma(F)$ to represent the set of all possible syndromes with which the faulty set F can be consistent.