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High accuracy modeling of sharp wave fronts for hyperbolic problems

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ABSTRACT

In this paper, the arbitrary order derivative (ADER) schemes based on the generalized Riemann problem are proposed to capture shock waves and contact discontinuities by coupling ghost fluid method (GFM). The reconstruction technique for spatial derivatives at cell boundaries is presented by piece-wise smooth WENO interpolations which are used as initial states of the Riemann problems. A level set function is used to keep track of the location of wave fronts. The shock waves are pushed forward by shock speeds which are obtained by the Rankine–Hugoniot conditions, whereas the contact discontinuities are advanced by local fluid velocities. Numerical examples show that the presented scheme is suitable for capturing fine flow structures and has an accuracy comparable to other methods designed for traditional contact discontinuity.

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1. Introduction

The conservative form of hyperbolic conservation laws provides the best flow description. As is well known from the theorem of Godunov [1], when extended to strong shock fluids, many high-order linear schemes encounter difficulties and generate spurious oscillations near discontinuities or sharp gradients of the solution. These oscillations pollute the numerical solution and are thus highly undesirable. One may consider to sacrifice the strict conservation and to write the equations in non-conservative form. The flux-splitting methods are most widely used [2] for front-capturing approaches. In all capturing methods the discontinuities are smeared over a certain number of nodes, which leads to artificial, intermediate states.

The ADER schemes was first proposed by Toro and his collaborators for linear hyperbolic problems [3]. Later the ADER finite volume schemes based on the GRP have been widely extended to the frameworks of discontinuous Galerkin finite element methods and non-linear systems with source terms on unstructured meshes in two and three space dimensions [4–6]. Recently ADER has also been extended to the equations of magnetohydrodynamics by Balsara and collaborators [7,8]. The direct arbitrary-Lagrangian–Eulerian ADER-WENO finite volume scheme is applied to solve 3D hydrodynamics problems on unstructured tetrahedral grids in [9]. The direct arbitrary-Lagrangian–Eulerian (ALE) one-step ADER-MOOD finite volume schemes for the solution of nonlinear hyperbolic systems of conservation laws on moving unstructured triangular and tetrahedral meshes is presented in [10]. Furthermore, Ferrari et al. proposed a high order scheme for one-dimensional compressible multi-phase flows [11].

The paper is organized as follows. The ADER scheme for one dimensional hyperbolic system is presented in Section 2. In Section 3, the method for capturing contact discontinuities and shock waves is described in detail. At last, the

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comparative results for solving one and two dimensional systems are provided in Section 4 and concluding remarks are drawn in Section 5.

2. Brief description of ADER schemes

Let us consider nonlinear hyperbolic conservation system for inviscid compressible fluids

$$U_t + F_x(U) = 0, \tag{1}$$

where $U(x, t)$ is the vector of unknown conservative variables, $F(U)$ is physical flux vector in x coordinate direction. The component form for Eq. (1) is

$$\begin{pmatrix} \rho \\ \rho u \\ E \end{pmatrix}_t + \begin{pmatrix} \rho u \\ \rho u^2 + p \\ (E+p)u \end{pmatrix}_x = 0, \tag{2}$$

where ρ is the density, u is the velocity, p is the pressure, $E = 0.5\rho u2 + e$ is the total energy per unit volume that is the sum of the internal energy and the kinetic energy, $e = \frac{p}{\gamma-1}$ is the specific internal energy and γ is the ratio of specific heat.

We present a non-oscillatory finite volume (FV) scheme of arbitrary accuracy in space and time for solving nonlinear hyperbolic systems (1) on Cartesian grids. This method has been numerically investigated in [4,5]. Here let $I_i = [x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$ be a control volume (a computational cell) and $\Delta x = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}$, $\Delta t = t^{n+1} - t^n$. Integration of Eq. (1) over a space–time control volume of dimensions $I_i \times [t^n, t^{n+1}]$ produces the following one-step finite-volume scheme:

$$\bar{U}_i^{n+1} = \bar{U}_i^n - \frac{\Delta t}{\Delta x}\left(F_{i+1/2} - F_{i-1/2}\right), \tag{3}$$

where $\bar{U}_i^n$ is the cell average of the solution at time level t^n, namely,

$$\bar{U}_i^n = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} U(x, t^n)dx \tag{4}$$

and $F_{i+\frac{1}{2}}$, the spatial average of physical fluxes over cell faces, is given by

$$F_{i+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} F\left(U(x_{i+1/2}, t)\right)dt. \tag{5}$$

The spatial and temporal accuracy of the final scheme depends merely on the order of the numerical flux $F_{i+1/2}$. In the following, we evaluate the ADER numerical flux $F_{i+1/2}$ in Eq. (5).

We reconstruct the point-wise values and all derivatives up to $r-1$ order from cell averages at the cell boundary $(x_{i+1/2}, 0+)$ by WENO interpolations which is a non-linear solution adaptive reconstruction. First, we perform one dimensional WENO sweep in the x coordinate direction and obtain left and right Godunov state of U and its x-derivatives. After the reconstruction is carried out for each point $(x_{i+1/2}, 0+)$ at the cell face we pose the generalized Riemann problem and obtain a high order approximation to $U(x_{i+1/2}, \tau)$. For simplicity, we only discuss the evaluation of the numerical flux $F_{i+1/2}$. According to Taylor series expansions in time, we have

$$U(x_{i+1/2}, \tau) = U(x_{i+1/2}, 0_+) + \sum_{k=1}^{K-1}\left(\frac{\partial^k U(x_{i+1/2}, 0_+)}{\partial \tau^k}\right)\frac{\tau^k}{k!} + O(\tau^K). \tag{6}$$

The leading term $U(x_{i+1/2}, 0_+)$ can be obtained by solving

$$\partial_t U + \partial_x F(U) = 0, \tag{7}$$

$$IC: U(x, 0) = \begin{cases} U_L(0) = U_L(x_{i+1/2}) & \text{if } x < x_{i+1/2} \\ U_R(0) = U_R(x_{i+1/2}) & \text{if } x > x_{i+1/2} \end{cases} \tag{8}$$

In order to obtain higher order Godunov states in x derivatives, we need to solve the Riemann problem with piece-wise constant data at $x = x_{i+1/2}$, $\tau = 0+$. We convert time derivatives to functions of spatial derivatives by means of Cauchy–Kowalewski procedure, see [4] and references therein, namely,

$$\partial U_t^{(k)}(x, t) = P^{(k)}\left(\partial U_x^{(0)}, \partial U_x^{(1)}, \ldots, \partial U_x^{(k)}\right) \tag{9}$$

where $\partial U_x^{(k)} \equiv \partial^k U(x, t)/\partial x^k$, $k = 1, \ldots, K-1$. The k-order spatial derivative $\partial_x^{(k)} U(x, t)$ obeys by the following manipulation of the PDEs:

$$\partial_t\left(\partial U_x^{(k)}(x, t)\right) + A_{i+1/2}(U)\partial_x\left(\partial U_x^{(k)}(x, t)\right) = H^{(k)}\left(\partial U_x^{(0)}, \partial U_x^{(1)}, \ldots, \partial U_x^{(K-1)}\right) \tag{10}$$

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