Applied Mathematics and Computation 330 (2018) 93–102

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Ergodic property of a Lotka–Volterra predator–prey model with white noise higher order perturbation under regime switching☆

Li Zu[a], Daqing Jiang[b,c,*], Donal O'Regan[d], Tasawar Hayat[c,e], Bashir Ahmad[c]

[a] College of Mathematics and Statistics, Hainan Normal University, Hainan 571158, PR China
[b] School of Science, China University of Petroleum (East China), Qingdao 266580, PR China
[c] Nonlinear Analysis and Applied Mathematics(NAAM)-Research Group, King Abdulaziz University, Jeddah, Saudi Arabia
[d] School of Mathematics, Statistics and Applied Mathematics, National University of Ireland, Galway, Ireland
[e] Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan

ARTICLE INFO

Keywords:
Ergodic property
Stationary distribution
Lotka–Volterra predator–prey model
Higher order perturbation

ABSTRACT

In this paper, we investigate a classical Lotka–Volterra predator–prey model with telephone noise and a higher order perturbation of white noise. The existence of a unique positive solution is discussed and sufficient conditions for the existence of an ergodic stationary distribution is established. Some simulation figures are presented to illustrate the analytical findings.

1. Introduction

For a Lotka–Volterra predator–prey model

$$\begin{cases} \dot{x}(t) = x(t)\big(a - by(t)\big), \\ \dot{y}(t) = y(t)\big(-c + hx(t)\big), \end{cases} \tag{1.1}$$

where a, b, c, h are positive constants notice $h\dot{x}(t) - a\frac{d[\log y(t)]}{dt} = ac - bhx(t)y(t)$ and $b\dot{y}(t) - c\frac{d[\log x(t)]}{dt} = -ac + bhx(t)y(t)$, and the system (1.1) has a periodic orbit $hx(t) - a\log y(t) + by(t) - c\log x(t) = C$ (here C is an arbitrary constant). If there is no influence from the environment, the population of system (1.1) develops periodically [1,2]. However environmental noise is an important component in an ecosystem (see e.g. [21,22]) and noise influences an ecological system in a variety of ways. Consider a popular type of environmental noise, namely white noise or Brownian motion (white noise analysis was initiated by Hida [25]). Environmental noise has the potential to have a huge impact on the population dynamics of a system, for example sufficiently large white noise can cause a population that would otherwise explode or tend to a unique endemic equilibrium to die out [28]. In the literature many authors have investigated population systems subject to white noise and dynamic properties such as existence of positive solutions, stochastic persistence, stationary distribution and extinction

☆ The work was supported by Program for NSFC of China (No: 11371085), the Fundamental Research Funds for the Central Universities (No: 15CX08011A), the Foundation of Hainan Provincial Science and Technology Department (No: 20161006) and Hainan province colleges and universities scientific research projects (No: Hnky2017ZD-14).

* Corresponding author at: School of Science, China University of Petroleum (East China), Qingdao 266580, PR China.
E-mail addresses: daqingjiang2010@hotmail.com, jiangdq067@nenu.edu.cn (D. Jiang).

https://doi.org/10.1016/j.amc.2018.02.035
0096-3003/

were examined (see [3–10,12–18]). Environment fluctuations manifest themselves mainly as fluctuations in the intrinsic growth rate of the prey population and in the death rate of the predator population [20], and in this paper, we assume the parameters a and $-c$ of system (1.1) are perturbed by $a \to a + \alpha x(t)\dot{B_1}(t),\ -c \to -c + \beta y(t)\dot{B_2}(t)$, where $B_1(t)$ and $B_2(t)$ are one-dimensional standard Brownian motion with $B_i(0) = 0\ (i = 1, 2)$ (here α^2 and β^2 are the intensities of the white noises). As a result, system (1.1) becomes a stochastic differential system

$$\begin{cases} dx(t) = x(t)[a - by(t)]dt + \alpha x^2(t)dB_1(t), \\ dy(t) = y(t)[-c + hx(t)]dt + \beta y^2(t)dB_2(t). \end{cases}$$

Also we note that the birth and death rates of some species in the dry season will be much different from those in the rainy season. Likewise, the carrying capacities often vary according to the changes in nutrition and/or food resources. They are random discrete events that happen at random epochs. A convenient formulation here is to use a continuous-time Markov chain taking values in a finite set and the resulting dynamic system is the so-called regime switching differential equations (that is, with colored noise, say Telegraph noise); we refer the reader to [3–5,7–9,11–16,19,24,26]. We introduce regime switching with the continuous-time Markov chain $\{r(t), t \geq 0\}$ taking values in a finite state space $\mathbb{S} = \{1, \ldots, N\}$, and system (1.1) can be expressed as

$$\begin{cases} dx(t) = x(t)[a(r(t)) - b(r(t))y(t)]dt + \alpha(r(t))x^2(t)dB_1(t), \\ dy(t) = y(t)[-c(r(t)) + h(r(t))x(t)]dt + \beta(r(t))y^2(t)dB_2(t). \end{cases} \tag{1.2}$$

Takeuchi et al. [11] studied system (1.2) with no white noise and discussed periodic orbits in different environmental regimes. Using white noise higher order perturbation, Luo and Mao [12] showed system (1.2) has a unique positive solution and they considered the stochastically ultimately boundedness and the boundedness of moment average in time. The authors in [23] studied periodic solutions and the stationary distribution of stochastic SIR epidemic models and in [4] the authors introduced a stochastic logistic model under regime switching II and considered its stochastic permanence. In [18] and [19] the authors studied a stochastic Logistic equation under regime switching and discussed global asymptotical stability and nonpersistence and stochastic permanence. However, there are currently no results concerning the stationary distribution and the ergodic property of system (1.2). There are a few papers in the literature which study the stationary distribution of population models affected by both white noise and regime switching (see [3,4,7,9,24]). Note the stationary distribution can be viewed as weak stability. Using ergodic theory given in [13,14], Zu et al. [7] considered a Lotka–Volterra stochastic predator–prey model having colored noise, Liu et al. [9] and Settati and Lahrouz [3] considered a stochastic Lotka–Volterra mutualistic system under regime switching, and obtained the ergodic property and positive recurrence. The authors in [24] investigated a stochastic Susceptible-Infective epidemic model under regime switching and they obtained the threshold between extinction and the existence of the stationary distribution. To the best of our knowledge there are no related results on the ergodic property and positive recurrence of the Lotka–Volterr predator–prey model affected by both Markov switching and white noise higher order perturbation. In this paper, we will discuss the existence of an ergodic stationary distribution of system (1.2). We assume that, for each $k \in \mathbb{S}$, $a(k), b(k), c(k), h(k), \alpha(k), \beta(k)$ are positive constants.

Section 2 surveys some necessary properties of a Markov process and a result on ergodicity and stationary distributions. Section 3 gives sufficient conditions for the existence of stationary distribution and the ergodic theory on stochastic system (1.2) under regime switching. In Section 4, we present some figures to illustrate our main results.

2. Preliminaries

Throughout this paper, unless otherwise specified, let $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathscr{F}_0$ contains all P-null sets). We denote by R^2_+ the positive zone in R^2, i.e., $R^2_+ = \{(x, y) \in R^2 : x > 0,\ y > 0\}$. Let $r(t)$, $t \geq 0$, be a right-continuous Markov chain on the probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathbb{P})$ taking values in a finite-state space $\mathbb{S} = \{1, \ldots, N\}$ with the generator $\Gamma = (\gamma_{uv})_{1\leq u,v\leq N}$ given by

$$\mathbb{P}\{r(t + \Delta t) = v | r(t) = u\} = \begin{cases} \gamma_{uv}\Delta t + o(\Delta t), & \text{if } u \neq v, \\ 1 + \gamma_{uv}\Delta t + o(\Delta t), & \text{if } u = v, \end{cases}$$

where $\Delta t > 0$ and γ_{uv} is the transition rate from u to v and $\gamma_{uv} \geq 0$ if $u \neq v$, while $\gamma_{uu} = -\sum_{u\neq v}\gamma_{uv}$. Almost every sample path of $r(t)$ is a right-continuous step function with a finite number of sample jumps in any finite subinterval of $R^+ = [0, \infty)$ [27]. There is a sequence $\{\tau_i\}_{i\geq 0}$ of finite-valued $\mathscr{F}_t$-stopping times such that $0 = \tau_0 < \tau_1 < \ldots < \tau_i \to \infty$ a.s. and

$$r(t) = \sum_{i=0}^{\infty} r(\tau_i)\mathbf{1}_{[\tau_i, \tau_{i+1})}(t),$$

where $\mathbf{1}_A$ denotes the indicator function of set A [26]. As a standing hypothesis, in this paper, we assume that the Markov chain $r(t)$ is irreducible and independent of the Brownian motion and assume $\gamma_{uv} > 0$ if $u \neq v$. From the classical theory of a Markov chain, the finite states imply the ergodic property and positive recurrence of $r(t), t \geq 0$. Hence the Markov chain has a unique stationary (probability) distribution $\pi = (\pi_1, \ldots, \pi_N) \in R^{1\times N}$ such that $\pi\Gamma = 0,\ \Sigma_{k=1}^N \pi_k = 1,\ \pi_k > 0,\ \forall k \in \mathbb{S}$. For any vector $e = (e(1), \ldots, e(N))^T$, let $\hat{e} = \min_{k\in\mathbb{S}}\{e(k)\}$ and $\check{e} = \max_{k\in\mathbb{S}}\{e(k)\}$.