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Evolutionary investors' power-based game on networks

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ABSTRACT

The classical prisoner dilemma game on networks ignores the heterogeneity of players that may lead to the remarkable differences of their payoffs in reality. With the consideration of the heterogeneity, we propose an investors' power-based game, where the payoffs of defectors depend on the efficiency of market and the related-power against cooperators. Economically, the efficiency of the market of investment is introduced in the game through a parameter α that becomes a key factor in the evolutionary process. Our simulation results show that, an improvement of efficiency benefits for the cooperation fundamentally. Furthermore, comparing with the result on BA scale-free networks, the evolution of cooperation performs great stability on WS small-world networks against the change of market efficiency. As the network of investment in real world may possess both of the properties of WS small-world networks and BA scale-free networks, the findings may be helpful in understanding and controlling the behaviors on the network of investment.

1. Introduction

Network is common in various fields from Internet to biology and society [1–10]. The neoclassical economic, as one of the most important subjects, emphasizes the individualism excessively, ignoring the interactions among agents [11–13]. Although several economists realized the importance of network in the economic researches [14–20], the network is introduced into economy widely until Granovetter [21]. Granovetter suggested that agents' irrationality will be transformed into rationality with the consideration of economical network. There is no doubt that network science promotes the development of economy. Since the social and economic network is characterized by small distance [22], high clustering [10,23] and unequal distribution of links [24,25], more empirical economic researches with networks are appeared, including the labor markets [26–29], technology markets [30,31] and international trades [32–34]. Especially, financial market is also focused by network science [35,36]. It is found that networks play essential roles in the risk share of financial markets. Recently, the microscopic behaviors among firms have been studied [37–42], verifying the interactions between companies is worthwhile to be discussed. Totally, the effect of networks, representing the relationships among the agents, should not be neglected.

Meanwhile, game theory, as the major method to dissect the behaviors of interactions, increases the flexibility in the network researches [43]. Remarkably, Nash [44] proposed Nash equilibrium that is a milestone of game theory, who believed a large number of repeated interactions from networks could form the equilibrium. With the inspiration, more and more scholars begin to study evolutionary games when Axelrod and Hamilton [45] presented the groundwork on repeated

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games. Nowak and May firstly consider the spatial prison dilemma game in [46]. Perc and Szolnoki make a comprehensive review on evolutionary games in recent years [47]. Within this evolutionary framework, the evolution process of cooperative behaviors is focused on. Some studies survey emergency of cooperative under different kinds of situations [48–50]. Most of academic researches start with two simple classic kinds of games, the prison dilemma game (PDG) and the snowdrift game (SG). The transformations of them illustrate many behaviors in the society [51–57]. In the original model, agents are supposed to have two strategies, such as cooperation and defection, which are the generalization of people's behavior in society [58–66]. Szolnoki, Perc [67]and Perc [68,69] consider the prisoner's dilemma on the random networks and scale-free networks respectively, in which the complex mechanisms in the real world are participated, such as distinguished players. This common framework of evolutionary game will be adopted in our research.

In neoclassical economics, most assumptions are suspected by researchers. On the one hand, agents in the market of neoclassical economy is of perfect rationality. It is so impracticable that the selection of rationality became a challenge to neoclassical economics [70–72]. Fortunately, the agents in the evolutionary dynamic are bounded rational, releasing the perfect rationality assumption in traditional economics [73]. The little change of assumptions created a great opportunity for the blend of economics and evolutionary games [74,75].

On the other hand, markets of neoclassical economics are always efficient [76]. According to a number of studies empirically, the market is often lack of efficiency in actual [77–79]. It reminds us that the efficiency of markets should be regarded as a key parameter in the economic model.

Motivated by these references, we explore the behaviors of investors against the background of investment markets. The powers of investors have effects on their payoffs of defection [80–87], reflecting their heterogeneity. The power-based game will be exhibited in Section 2. Section 3 gives the rule of strategy updating in evolutionary process. The relationship between the strategy and payoff of a single investor will be considered in Section 4. With the consideration of the features of investors' network, we examine the evolutionary dynamics of power-based game on BA scale-free networks and WS small-world networks respectively in Section 5. Finally, Section 6 concludes.

2. Power-based game

In the original spatial game model, payoffs for different pairs of players are the same, i.e. the same constant, while they adopt the same strategies. For example, in the evolutionary process of prisoner dilemma game (PDG), defectors in all D-C (defector-cooperator) pairs of network get the same highest payoff as his cooperator get nothing. However, agents in real network are heterogeneous in some aspects that may affect their payoffs.

Here, in the view of heterogeneity of players, we introduce the power-based game integrated with individuals' degrees. Degree is used to measure the power of nodes in the traditional theory of graph. Especially in investment networks, investors with higher degrees, i.e. with formidable connections, always have more powers because of their richer sources of messages [80–87].

Similar to the common scheme of prisoner dilemma game, the payoffs of our power-based game are written in a rescaled payoff matrix:

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} 1 & 0 \\ (d_i/d_j)^{\alpha} & 0 \end{pmatrix} \end{array}$$

where d_i, d_j are the degree of the defector i and the cooperator j respectively. α measures the efficiency of market, which lies between 0 and 1. As mentioned above, the power-based game is set in the investment networks. Two investors will receive the same reward that equal to 1 if they both cooperate. But once they make contrary strategies, the defector can exploit the cooperator and obtain the payoff $(d_i/d_j)^{\alpha}$, while the cooperator gets nothing. In the D-C pair, the defector's payoff depends on the fraction of powers of his opponent and himself, i.e. depends on the fraction of degrees of his opponent and himself.

Obviously, if the defector i's power is stronger than the cooperator j's, which means $d_i > d_j$, the defector will gain more than 1. In other words, for the higher-degree agent, defection is dominant. Generally, given the cooperated rival, defection is not sure to be dominant, which is different from the prisoner dilemma game (PDG). If the defector i's power is weaker than the cooperator j's, which means $d_i < d_j$, defector will get less than 1. Apparently, the rational smaller-degree agent must select cooperation. Therefore, the heterogeneity of players is embodied in the scheme of power-based game, through the degrees of players.

The further explanation about α is as follow. From the mathematical perspective, α controls the scale of the defector's payoff. Besides, economically, α reflects the efficiency of investment market. Traditionally, the efficiency of investment market is illustrated as its digestive ability of information. In the weak-form efficient market, investors with the stronger powers always implement arbitrage successfully on the basis of his richer sources of messages. The weaker efficiency of market corresponds to that α increases to 1 in our model. In this situation, payoffs will be affected seriously by the heterogeneous powers of investors.

Instead, arbitrage opportunities by means of richer messages will hardly exist in a more efficient market, also called the strong-form efficient market. In other words, the diversity of investors' powers is weakened because the market has included