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Solving a class of singular two-point boundary value problems using new effective reproducing kernel technique

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ABSTRACT

Based on reproducing kernel theory, an efficient reproducing kernel technique is proposed for solving a class of singular two-point boundary value problems with Dirichlet boundary conditions. It is implemented as a new reproducing kernel method. In this method, reproducing kernels with Chebyshev polynomials form are used. Convergence analysis and an error estimation for the method in L^2_w space are discussed. The numerical solutions obtained by the method are compared with the numerical results of reproducing kernel method (RKM). The results reveal that the proposed method is quite efficient and accurate.

1. Introduction

The singular boundary value problems have been investigated in a wide variety of problems in science and engineering such as theory of thermal explosions, studies of Electro-hydrodynamics, nuclear physics, gas dynamics, chemical reaction, studies of atomic structures and atomic calculations. In general, classical numerical methods are not applicable to the singular two-point BVP, because of the singularity. However, there are non-classical approaches available for solving some classes of singular two-point boundary value problems numerically [2–4,7–13]. In this paper, based on reproducing kernel with polynomial form [1], we propose an effective numerical technique for sol ving singular two-point BVPs of the following form:

$$\begin{cases} x''(t) + \dfrac{1}{p(t)}x'(t) + \dfrac{1}{q(t)}x(t) = f(t), & a < t \le b, \\ x(a) = A, \quad x(b) = B, \end{cases} \tag{1.1}$$

where $f \in L^2_w[a,b]$ and $p(t) = O\big((t-a)^{\alpha}\big), q(t) = O\big((t-a)^{\beta}\big)$ for $0 < \alpha,\ \beta \le 1$ are sufficiently regular given functions such that Eq. (1.1) has a unique solution and A, B are finite constants.

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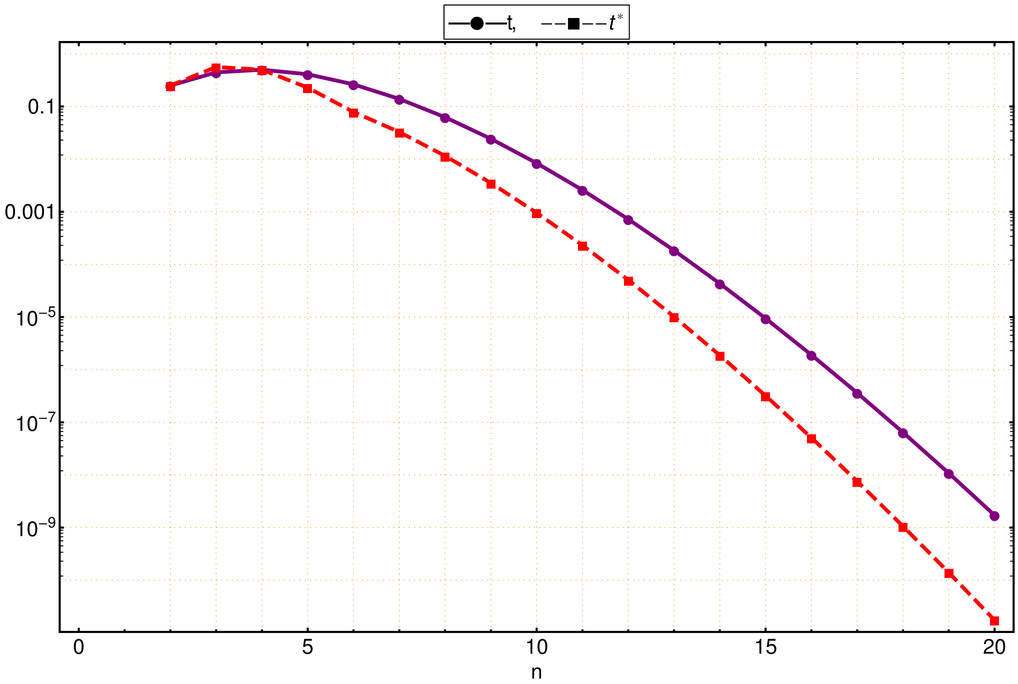

Fig. 1. The sequence $\{4^{n-3}\hbar_n^{n-1}\}_{n=2}^{20}$ for $t_{ni} = \frac{i+2}{n}$ and $t^*_{ni} = \frac{1}{2}\left(cos(\frac{\pi i}{2n}) + 1\right)$.

Table 1
Numerical results for Example 1.

t	Absolute errors: $\lvert r_n\rvert$		
	C-RKM		RKM [2]
	$n = 2$	$n = 3$	$n = 51$
0.001	1.66407E−03	7.2491E−16	5.8E−12
0.08	1.1678E−01	4.3872E−14	4.8E−07
0.16	2.0250E−01	3.3782E−14	2.7E−06
0.32	2.9304E−01	4.9960E−14	1.3E−05
0.48	2.9619E−01	3.7886E−15	2.9E−05
0.64	2.3654E−01	5.9813E−14	4.1E−05
0.80	1.3867E−01	8.0769E−14	3.9E−05
0.96	2.7136E−02	2.8831E−14	1.1E−05
1.00	0	0	0
$\lvert\lvert r_n\rvert\rvert_{\circ L^2_w}$	2.2725E−01	6.0533E−14	*

2. Construction of reproducing kernels with Chebyshev polynomials

Denote by $P_n[a, b]$ the set of all polynomials on $[a, b]$ with real coefficients and degree less than or equal to n, namely,

$$P_n[a, b] := span\left\{1, t, t^2, \dots, t^n\right\},$$

equipped with the following inner product

$$\langle x, y\rangle_{P_n} = \int_a^b \omega(t)x(t)y(t)dt, \quad \forall x, y \in P_n[a, b], \tag{2.1}$$

and the norm

$$||x||_{P_n} = \sqrt{\langle x, x\rangle_{P_n}}, \quad \forall x \in P_n[a, b],$$

where ω is a positive weight function.

For positive weight function $\omega(t) = \frac{1}{\sqrt{1-(\frac{2t-a-b}{b-a})^2}}$, the sequence of shifted Chebyshev polynomials of the first kind in t are defined on $[a, b]$ and can be determined as follows:

$$T^*_i(t) = 2\left(\frac{2t - b - a}{b - a}\right)T^*_{i-1}(t) - T^*_{i-2}(t), \quad i \geq 2, \tag{2.2}$$