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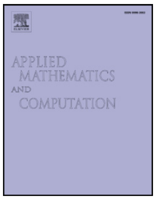

Pattern formation in a ratio-dependent predator-prey model with cross-diffusion

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ABSTRACT

In this paper, a ratio-dependent predator-prey model with cross-diffusion is studied. By the linear stability analysis, the necessary conditions for the occurrence of Turing instability are obtained. Moreover, the amplitude equations for the excited modes are gained by means of weakly nonlinear analysis. Numerical simulations are presented to verify the theoretical results and show that the cross-diffusion plays an important role in the pattern formation.

1. Introduction

Since the Lotka–Volterra predator-prey model was proposed by Lotka [1] and Volterra [2] independently, many predator-prey models have been proposed by the researchers to describe the predator-prey interactions in the real world. Since the importance of the predator-prey model, the research on its dynamics has one of the most challenging areas of the population ecology and many theoretical results have been obtained in the past decades. The following ratio-dependent predator-prey system

$$\begin{cases} \frac{dN}{dt} = rN(1-\frac{N}{K}) - \frac{\alpha NP}{P+\alpha\beta N}, \\ \frac{dP}{dt} = \frac{\eta\alpha NP}{P+\alpha\beta N} - \gamma P, \end{cases} \tag{1}$$

was introduced by Arditi and Ginzburg [3]. Here N and P stand for the prey and predator densities, respectively. r is the prey intrinsic growth rate, K is the environmental carrying capacity, α is the predator's attack rate, β is the handling time, η accounts for the efficiency of biomass conversion from the predation and γ is the predator's per-capita death rate. For a detailed explanation of (1), please refer to [3,4] and the explanation therein.

On the other hand, reaction-diffusion systems arising in a wide range of fields, including Biology, Chemistry, Physics and Ecology, have been widely used to study the mechanism of pattern formation [5–20]. These researches have shown that a spatial dispersion plays an important role in the pattern formation. Introducing the spatial dispersion into (1), Wang et al. [10] and Song and Zou [4] investigated the following diffusive ratio-dependent predator-prey system

$$\begin{cases} \frac{\partial N(x,t)}{\partial t} = rN(x,t)(1-\frac{N(x,t)}{K}) - \frac{\alpha N(x,t)P(x,t)}{P(x,t)+\alpha\beta N(x,t)} + d_{11}N_{xx}(x,t), \\ \frac{\partial P(x,t)}{\partial t} = \frac{\eta\alpha N(x,t)P(x,t)}{P(x,t)+\alpha\beta N(x,t)} - \gamma P(x,t) + d_{22}P_{xx}(x,t), \end{cases} \tag{2}$$

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where $d_{11} \geq 0$ and $d_{22} \geq 0$ are the self-diffusion coefficients for the prey and predator, respectively.

Introducing the dimensionless variables

$$u = \frac{\alpha\beta}{\eta K}N, \quad v = \frac{\alpha\beta}{\eta^2 K}P, \quad \widehat{t} = \frac{\eta}{\beta}t, \quad \widehat{x} = \sqrt{\frac{\eta}{\beta}}x,$$

and then dropping the hats for the simplicity of notations, system (2) can be transformed to

$$\begin{cases} \frac{\partial u}{\partial t} = au(1 - \frac{u}{b}) - \frac{buv}{bu+v} + d_{11}u_{xx}, \\ \frac{\partial v}{\partial t} = (\frac{bu}{bu+v} - c)v + d_{22}v_{xx}, \end{cases} \tag{3}$$

where $a = \frac{r\beta}{\eta}, b = \frac{\alpha\beta}{\eta}, c = \frac{\gamma\beta}{\eta}$. And the parameters a, b and c are all positive.

For system (3), Wang et al. [10] obtained the condition of Hopf, Turing and wave bifurcation and showed that the spatially extended model had more complex dynamics patterns (stripe or spot or coexistence of both) by numerical simulations. Song and Zou [4] have also studied the stability of the positive constant equilibrium, Turing instability, Hopf and steady state bifurcations. In addition, many researchers have also studied similar systems to (3). For instance, Banerjee and Abbas [21] considered the global bifurcation for a ratio-dependent predator-prey model with death rate for the predator at first, then by incorporating diffusion terms, the varieties of stationary and non-stationary spatial patterns were investigated. Liu [22] presented a diffusive predator-prey model with ratio-dependent functional response, and found that predation rate played an important role in pattern formation. The mathematical analysis and numerical simulations showed that transition from stationary patterns to non-stationary patterns as long as the predation rate varied. Camara et al. [23] analyzed the spatial pattern formation of a diffusive predator-prey system with ratio-dependent functional response, they determined the appropriate condition of Turing instability around the interior equilibrium point. The self-replication pattern formation in the spatial-temporal prey-predator model with ratio-dependent functional response were reported by Banerjee [24].

Most of the above researchers were interested in the Turing instability and pattern caused by self-diffusion. However, both experimental and theoretical studies also show the effect of the cross-diffusion on the pattern formation. For instance, the experiments have revealed that cross-diffusion coefficients in the BZ-AOT system are quite significant [25,26]. Peng and Zhang [27] have shown that cross-diffusion can also result in pattern formation for a predator-prey system with Allee effect. Gambino et al. [28] have investigated the complex dynamics originated by a cross-diffusion-induced subharmonic destabilization of the fundamental subcritical Turing mode in a predator-prey reaction–diffusion system. In this paper, we are interested in the effect of the cross-diffusion on system (3), i.e., we consider the following system

$$\begin{cases} \frac{\partial u}{\partial t} = au(1 - \frac{u}{b}) - \frac{buv}{bu+v} + d_{11}\nabla^2 u + d_{12}\nabla^2 v, & (x, y) \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} = (\frac{bu}{bu+v} - c)v + d_{21}\nabla^2 u + d_{22}\nabla^2 v, & (x, y) \in \Omega, t > 0, \end{cases} \tag{4}$$

where Ω is the bounded domain in R^2, with a smooth boundary $\partial\Omega$, $d_{12}(d_{21})$ is the cross-diffusion coefficient, which denotes that the influence of the predator (prey) density to the prey (predator) density. When $d_{12} > 0$, the prey is repelled from the predator, and when $d_{12} < 0$, the prey is attracted. Meanwhile, d_{21} has the same meaning with the role of the prey and predator switched. In this paper, we always assume that $d_{12} > 0$ and $d_{21} > 0$.

For system (4), we use the following initial conditions

$$u(x, y, 0) = \varphi(x, y), v(x, y, 0) = \psi(x, y) \geq 0, x, y \in \Omega,$$

and the Neumman boundary conditions

$$\frac{\partial u}{\partial \mathbf{n}} = 0, \quad \frac{\partial v}{\partial \mathbf{n}} = 0,$$

where $\mathbf{n}$ is the external unit normal vector of the boundary $\partial\Omega$.

In this paper, we investigate the stability of the positive equilibrium, cross-diffusion-induced instability and spatial patterns for the predator-prey model (4). Compared with the works of [4] and [10], we are more interested in the effect of cross-diffusion term d_{21}. Theoretical analysis and numerical simulation show that the cross-diffusion leads to the appearance of pattern. This paper can be considered as the generalization of partial results in [4] and [10].

As for the study of biological pattern formation, we would also like to mention that models of cyclic dominance are traditionally employed to study biodiversity in biologically inspired settings and cyclic dominance is also at the heart of predator-prey interactions [29]. The pattern formation, spatiotemporal dynamics and important role of zealots in models of cyclic dominance have been widely studied in [29–32].

The rest of this paper is organized as follows. In Section 2, we investigate the stability and Turing instability of the positive equilibrium. The amplitude equations are derived by means of weakly nonlinear analysis in Section 3. In Section 4, numerical simulations show that the different types of patterns appear with the change of cross-diffusion coefficient d_{21}. Finally, we give a brief conclusion in Section 5.