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Burst ratio in the finite-buffer queue with batch Poisson arrivals

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ABSTRACT

We study the burst ratio in the queueing system with finite buffer and batch arrivals. The study is motivated by computer networking, in which packet losses occur due to queueing mechanisms and buffer overflows. First, we derive the formula for the burst ratio in the case of compound Poisson arrivals, general distribution of the service time and general distribution of the batch size. Then, we study its asymptotic behavior, as the buffer size grows to infinity. Using the obtained analytical solutions, we present several numerical examples with various batch size distributions, service time distributions, buffer sizes and system loads. Finally, we compare the computed burst ratios with values obtained in simulations.

1. Introduction

Roughly speaking, the burst ratio, [1], is a characteristic of the packet loss process, informing us about the inclination of losses to occur in long series. It is defined as the average length of series of packets lost one after another, divided by the average length of series of packets lost in the Bernoulli process (i.e. with independent losses).

Formally speaking, consider a sequence of binary random variables $Y_1, Y_2, Y_3, \dots,$ perhaps mutually dependent, each one assuming values 0 and 1 with probabilities L and $1 - L$, respectively. In networking, variables $Y_1, Y_2, Y_3, \dots$ symbolize consecutive packets passing through a network node, where a packet transmitted correctly to the next node is marked by 1, while a packet lost at the node—by 0. Obviously, parameter L is the loss probability (or the long-run packet loss ratio).

Let $\overline{G}$ denote the average length of series of consecutive zeroes in sequence $Y_1, Y_2, Y_3, \dots$. Note that, even for a small value of L (e.g. 0.01), the value of $\overline{G}$ can be arbitrary large (e.g. 500), due to possible strong dependencies between variables Y_i.

On the other hand, consider a Bernoulli sequence $Z_1, Z_2, Z_3, \dots,$ i.e. a sequence of binary random variables, with probability of 0 equal to L in each step, and with all variables $Z_1, Z_2, Z_3, \dots$ being mutually independent. Let $\overline{K}$ denote the average length of series of consecutive zeroes in sequence $Z_1, Z_2, Z_3, \dots$.

The burst ratio, B, of sequence $Y_1, Y_2, Y_3, \dots$ is defined as:

$$B = \frac{\overline{G}}{\overline{K}}. \tag{1}$$

Of course, the same packet loss probability is assumed in both the nominator and denominator, whether the average length of consecutive zeroes has been calculated from measurements or from a model.

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A trivial verification shows that

$$\overline{K} = \frac{1}{1-L}.$$

Thus, we can rewrite (1) as:

$$B = \overline{G}(1-L). \tag{2}$$

Naturally, the value of B greater than 1 indicates that the losses have a higher tendency to occur in long series, than in the case of random, independent loss. The value of B smaller than 1 indicates that the losses are more scattered, than in the case of random loss.

In computer networking, it is strongly preferred to have small burst ratios. This is due to the fact that long series of consecutive packet losses deteriorate significantly the quality of real-time multimedia transmissions, e.g. the Internet telephony, Internet television, etc. Namely, it is far better if the sequence of delivered-lost packets looks like this:

11110111101111101111011111011101111101111

rather than like this:

11111111111100011111111111000011111111111

even though in the former sequence there is exactly the same number of losses, as in the latter. (For a deeper discussion of the influence of the burst ratio on the quality of the digitalized voice transmissions, we refer the reader to [2,3]. In particular, formula (7–29) of [3] present directly the impairment of the voice quality as a function of B.)

Now, one of the most important reasons of packet loss in networking, especially in TCP/IP networks, is queueing of packets in network nodes (routers). Namely, the incoming packets are stored in a buffer, before transmission via the output interface. If this buffer is full, the arriving packets are deleted. In real networks, it happens all the time.

For this reason, it is natural to study the burst ratio in the finite-buffer queueing model, which mimics precisely the actual reason of losses—buffer overflow events. This is what we do in this paper.

In the study, we assume general distribution of the service time. This makes the formulas more complicated, but allows for precise modeling of the packet transmission time. In a TCP/IP network, the packet sizes may vary according to some distribution. This distribution translates then to the distribution of the service (transmission) time.

To make the model even more accurate, we enable the possibility of arriving of packets not only as single units, but also in groups (batches). This is motivated by the fact that a significant fraction of traffic in contemporary networks is carried using the TCP protocol. It is well known that hosts exploiting the TCP protocol have the tendency to inject the packets into the network in groups. This is caused by the design of the protocol, which allows sending a whole group of W packets, before receiving a confirmation of the delivery of the previously sent packets. (More information on the batch structure of the TCP traffic and its modeling can be found e.g. in [4].)

It is intuitively clear that the batch structure of the traffic may have a deep impact on the burst ratio. Namely, when the buffer is full and a group of packets arrive, the whole group is lost (instead of a single packet, as it would happen in the case of the single-arrival traffic). Therefore, the value of $\overline{G}$ is typically much higher for the batch-arrival traffic. Using numerical examples, we will confirm this intuition in Section 6.

Summarizing, in this paper we study the burst ratio in the queueing system with finite buffer, in which losses of packets are caused by buffer overflows. To model the traffic, the batch Poisson process is used, with the distribution of the batch size in general form. Similarly, the service time distribution is general.

In Section 2, we characterize the previous work on the burst ratio. Then, Sections 3 and 4 present the main results of the paper. Namely, in Section 3, the exact formula for the burst ratio, depending on the arrival rate, the batch size distribution, the service time distribution and the buffer size, is derived (Theorem 1). In Section 4, the limiting formula for the burst ratio, as the buffer size grows to infinity, is derived (Theorem 2). It can be used often as a simple approximation of the burst ratio value, even for relatively small buffers. In Section 5, using numerical examples, we demonstrate the impact of the buffer size, the average batch size, the variance of the batch size, the service time distribution, and the system load, on the burst ratio. We present simulation results, obtained in various scenarios, each confirming the accuracy of the analytical results. We also show the simplified formulas for the special case—the $M/M/1/N$ queue. Finally, conclusions are presented in Section 6.

2. Previous work

We are not aware of any published work on the burst ratio in queueing models of any type.

The simplest and most important characteristic of the loss process is the loss probability (loss ratio), L. Several papers have been published to date on its actual measurements in computer networks (see, e.g. [5–9]), as well as its analytical computations based on queueing models (see, e.g. [10–12]). The analytical papers differ in assumptions about the arrival process. In [10], the Poisson process is used; in [11], the Markov-modulated Poisson process is exploited, while in [12], the batch Markovian arrival process.

As for the burst ratio, the previous work on this characteristic was based on modeling packet losses via some stochastic processes, which were not originated in queueing mechanisms. In most papers, Markov chain models of the loss process