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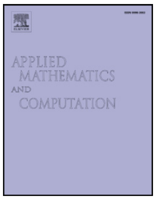

Graph-theoretic approach to synchronizing stochastic coupled systems with time-varying delays on networks via periodically intermittent control

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ABSTRACT

This article concerns the inner exponential synchronization (IES) problem of a class of stochastic coupled systems with time-varying delays on networks (SCSTDNs). Periodically intermittent control is imposed on SCSTDNs to realize IES. By combining graph theory with Lyapunov method, a Lyapunov function is constructed and some sufficient conditions guaranteeing IES of SCSTDNs under periodically intermittent control are derived. Additionally, the analysis of IES for a stochastic coupled oscillators model with time-varying delays is performed to show the applicability of the analytical results. Finally, an example with numerical simulation is presented to illustrate the validity of our theoretical results.

1. introduction

Over the past decades, coupled system on networks (CSNs), as a special kind of complex networks, have been studied extensively because they have many important applications in various areas such as moving image processing, optimization, speed detection of moving subjects, secure communication and so on, see [1–4] and references therein. Especially, the synchronization phenomena in CSNs have secured growing attention among the research community since the fact that synchronization can well explain many natural phenomena and also has lots of potential applications in chemical reactions, biological systems, information science and so on [5–7]. As a result, various kinds of synchronization problem of CSNs have been studied and numerous results has been proposed [8,9]. For instance, in [9], tang et al. discussed pin cluster synchronization on complex dynamical networks in Lur'e Forms. It should be pointed out that synchronization of all the nodes within a network occupies a pivotal and vital position in concrete application. Therefore, rapidly growing research interests have focused on the synchronization problem of CSNs, and a wide variety of synchronization criteria have been presented [10–12].

In general, control techniques have great influence on the realization of network synchronization. Hitherto, various control schemes have been used to study the synchronization of CSNs, such as state feedback control [13], impulsive control [14], pinning control [15] and intermittent control [16]. From the viewpoint of engineering applications, the control cost of continuous feedback control is expensive. Compared with continuous feedback control of synchronization, impulsive control and intermittent control are discontinuous control methods. The main idea of impulsive control is to change the states of continuous dynamic systems at certain time moments, while the intermittent control is to control systems via discontinuous

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control inputs at a time duration which is usually periodic. Hence, they can dramatically reduce the control cost and the amount of the transmitted information. Additionally, since the states of controlled systems are changed directly, impulsive control is an effective approach when the states are observable. But it seems to be invalid when the states of controlled systems are unobservable [16]. What's more, in comparison with impulsive control, periodically intermittent control is easier to be implemented in practice due to it has a nonzero control time. Indeed, intermittent control arises naturally in a wide variety of applications, such as ecosystem management, windshield wipers intermittent control, and control of hyper uricacidemia in the treatment of gout. Owing to those merits, periodically intermittent control have been applied successfully to study the synchronization problem of various kinds of coupled networks [17–19]. For instance, in [17], the synchronization problem for a class of complex delayed dynamical networks was investigated by pinning periodically intermittent control. In [19], some exponential synchronization criteria for coupled memristor-based chaotic neural networks with time-varying delays were derived via periodically intermittent control.

In real applications, CSNs are inevitably subject to various types of environmental noises which exist in external environment. Hence, one should analyze a more generalized model by considering the stochastic phenomena [20–22]. In [21], Yan et al. successfully studied the quantitative exponential stability and stabilisation problems of discrete-time Markov jump systems with multiplicative noises. On the other hand, due to the finite speed of information transmission and traffic congestions among the nodes, the synchronization problem of CSNs with coupling delays has widely been studied in the literature. For example, the synchronization problem of hybrid coupled complex dynamical networks with mixed time delays was discussed in [23]. Hence, the synchronization problem of various types of networks with environment noise and time delays has attracted much attention from researchers in different fields [24–26]. In [25], the problem of stochastic synchronization analysis was investigated for a new array of coupled discrete-time stochastic complex networks with randomly occurred nonlinearities and time delays. Synchronization and state estimation problems were considered for an array of coupled discrete time-varying stochastic complex networks over a finite horizon in [26]. Therefore, to make model more general, it is necessary to take stochastic disturbance and time delays into account.

It is a meaningful work to study the synchronization of CSNs, but it is also formidable. On the one hand, some factors dramatically rise the complexity of the systems. It is known that time delays and environmental noises are often sources of chaos of CSNs. Moreover, synchronization of CSNs is also affected by the complex coupling structure. On the other hand, the main method for dealing to the synchronization problem of stochastic coupled systems with time-varying delays on networks (SCSTDNs) is essentially based on the search of common Lyapunov functions or variations of the same framework. However, it is very difficult to construct a proper Lyapunov function for SCSTDNs directly, this is also a disadvantage of the Lyapunov approach. Therefore, the synchronization analysis for SCSTDNs is quite involved and complex. It is inspiring that Michael Y. Li and his co-authors [1] developed a systematic approach that took advantage of graph theory to construct global Lyapunov functions when investigating stability of large-scale coupled ordinary differential systems. The graph-theoretical method can avoid the difficulty in directly constructing global Lyapunov functions for coupled systems. Lots of literature which indicate the graph-theoretical approach is applicable in kinds of coupled networks have been published and the graph-theoretical approach has been regarded as an important method for dynamical analysis of CSNs [27–31]. In [27], stability of a class of stochastic coupled systems with Lévy noise on networks was studied by using this graph-theoretical approach. Synchronization of stochastic coupled systems with feedback control based on discrete-time state observations was investigated via the graph-theoretical method in [28]. Although there have been many results on the problem of dynamical analysis for CSNs, there are nearly no results on the inner exponential synchronization (IES) issue of SCSTDNs under periodically intermittent control by utilizing the graph-theoretical technique.

Motivated by the aforementioned discussion, the aim of this paper is to study the IES problem of SCSTDNs under periodically intermittent control. The contributions are as follows. Firstly, independent time-varying delays influence the state of different vertices without the disturbance of white noise and influence the intension of white noise, respectively, are all taken into account. Secondly, by combing graph theory with Lyapunov method, a Lyapunov-type theorem is obtained to ensure IES of SCSTDNs under periodically intermittent control. After that, a coefficient-type theorem in terms of the coefficients of the formulated systems is derived on the basis of the Lyapunov-type theorem. Thirdly, we apply our theoretic results to stochastic coupled oscillators with time-varying delays to show the applicability of our main results. Finally, a numerical simulation is given to illustrate the validity of the proposed criteria.

The reminder of this paper are as follows. In Section 2, some preliminaries and a mathematical model of SCSTDNs are presented. Some sufficient conditions to ensure IES of SCSTDNs under periodically intermittent control are established in Section 3. In Section 4, we apply our theoretical results to stochastic coupled oscillators with time-varying delays to show the practicality of the theoretical results. A numerical example is provided in Section 5. Conclusion is finally drawn in Section 6.

2. Preliminaries and model formulation

In this section, we first introduce some basic notations and a vital lemma in graph theory. Then, we give the model formulation and finally present two important definitions.