Applied Mathematics and Computation 330 (2018) 266–287

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Designing Gabor windows using convex optimization

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ARTICLE INFO

Keywords:
Short time Fourier transform
Dual window design
Tight window design
Convex optimization
Gabor system

ABSTRACT

Redundant Gabor frames admit an infinite number of dual frames, yet only the canonical dual Gabor system, constructed from the minimal ℓ^2-norm dual window, is widely used. This window function however, might lack desirable properties, e.g. good time–frequency concentration, small support or smoothness. We employ convex optimization methods to design dual windows satisfying the Wexler–Raz equations and optimizing various constraints. Numerical experiments suggest that alternate dual windows with considerably improved features can be found.

1. Introduction

Filterbanks, in particular those allowing for perfect reconstruction (PR), are fundamental and essential tools of signal processing. Consequently, the construction of analysis/synthesis filterbank pairs forms central topic in the literature, relying on various approaches, like polyphase representation [1] or algebraic methods [2]. Other methods rely on frame theory [3], similar to the approach we wish to present. Probably the most widely adopted type of filterbank are modulated cosine and Gabor filterbanks (or transforms) [4–6], which are closely related. Gabor transforms, also known as sampled short-time Fourier transforms, provide a uniform time–frequency representation by decomposing a signal into translates and modulations of a single *window function*. They have been used in various applications, among others [7–10], and variations [11–13]. Such filterbanks have a rich structure, are easy to interpret and allow for efficient computation. A substantial body of work exists on the subjects of invertibility of Gabor filterbanks, perfect reconstruction pairs of Gabor windows and window quality, with a strong emphasis on the overcomplete case [14,15]. The ability of the analysis filterbank to separate signal components and the precision of the synthesis operation, after coefficient manipulation, depend crucially on the time and frequency concentration of the windows used. While either the analysis or synthesis window can be chosen almost freely, tuned to the desired properties such as optimal time and frequency concentration, choice of the *dual* window is restricted to the set of functions such that a PR pair is obtained. For computational reasons, detailed below, there is a canonical choice for the dual window, used almost exclusively. However, this *canonical dual window* might not be optimal with regards to the desired criteria, such as time–frequency concentration or short support, required for high quality processing and efficient computation respectively.

Therefore, a flexible method to compute optimal (or optimized) dual windows, considering the full set of possible choices and valid for any set of starting parameters, provides a valuable tool for the signal processing community. We obtain such a method by merging considerations from the theory of Gabor frames with the tools provided by modern convex optimization.

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https://doi.org/10.1016/j.amc.2018.01.035
0096-3003/

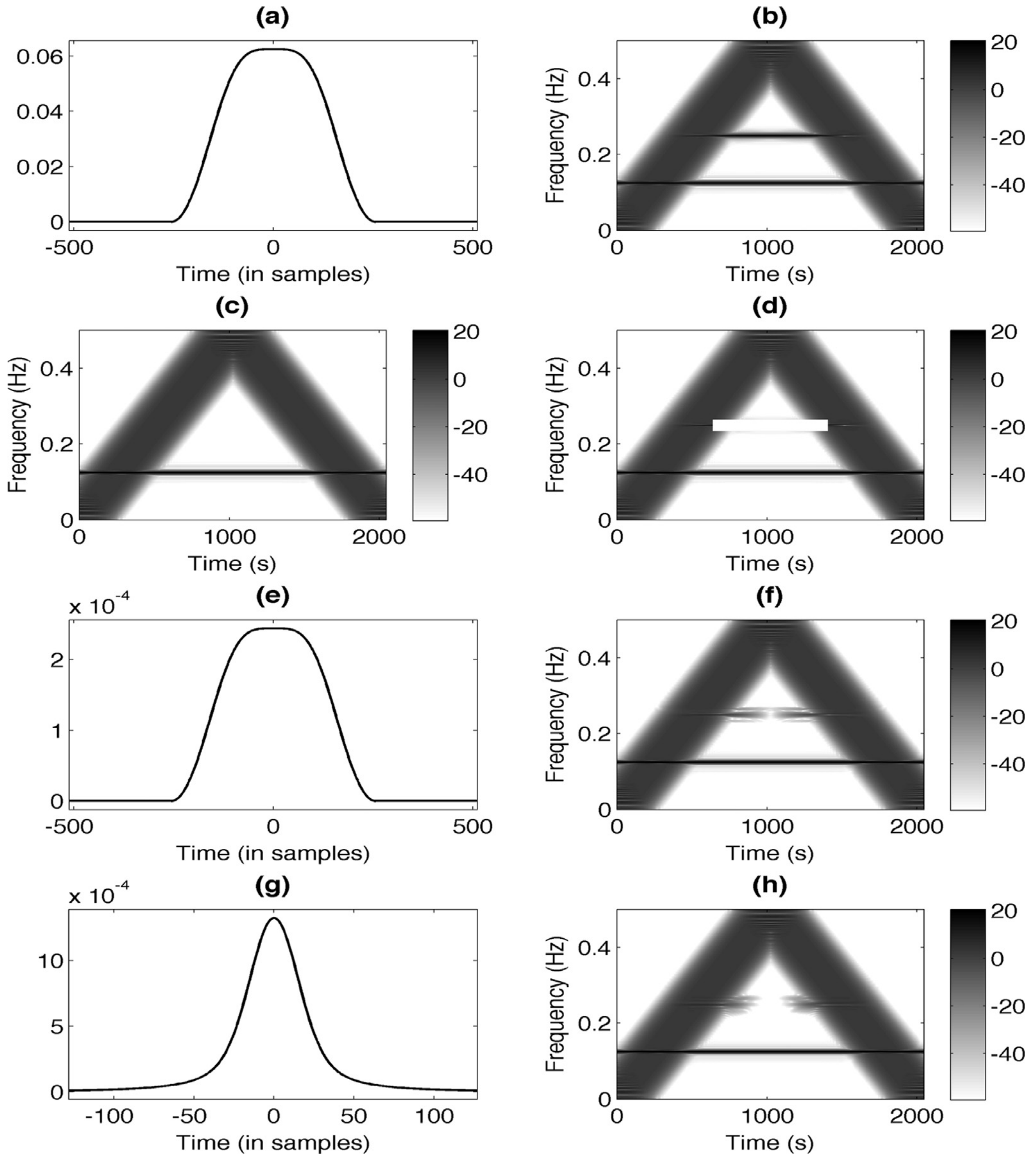

Fig. 1. Reconstruction from modified coefficients: (a) analysis window used to compute the spectrogram,[1] (b) spectrogram of a synthetic test signal, (c) unachievable 'oracle' target spectrogram, (d) modified spectrogram. (e, g) 2 different dual windows and (f, h) spectrograms after synthesis from modified coefficients. Note that the *smearing* effect depends on the concentration of the synthesis window.

The optimization framework we present is not limited to concentration or support optimization, but allows optimization with regards to any criterion that can be expressed through a suitable convex functional. Nonetheless, the aforementioned criteria are of universal importance and well-suited to demonstrate the capabilities and limitations of our method, which is why they form the focus of this contribution.

Since all dual windows perform perfect reconstruction from unmodified Gabor coefficients, the purpose of constructing alternative dual windows might not immediately be obvious beside the minimization of the support. However, if the coefficients are modified, e.g. through signal processing procedures such as frame multipliers [16–18], also known as Gabor filters [19], the shape of the dual window plays an important role in the quality and localization of the performed modifications. After processing, a signal is synthesized from the modified coefficients employing a dual Gabor filterbank. Let us illustrate the consequences of the window on the synthesis process after modification of the time–frequency representation with a toy example. For this example, we wish to remove an undesirable time–frequency component from a synthetic signal. In Fig. 1, we want to remove a localized sinusoid with only minor alteration of the remaining signal. This filtering operation relies on a joint time–frequency representation, since at each time or frequency position, several signal components

[1] The spectrogram is the squared magnitude of a Gabor-type TF representation.