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Media coverage campaign in Hepatitis B transmission model

Muhammad Altaf Khan[a,c,*], Saeed Islam[a], Gul Zaman[b]

[a] Department of Mathematics, Abdul Wali Khan University, Mardan, Khyber Pakhtunkhwa, Pakistan
[b] Department of Mathematics, University of Malakand, Chakdara Dir(L), Khyber Pakhtunkhwa, Pakistan
[c] Department of Mathematics, City University of Science and Information Technology, Khyber Pakhtunmkhwa, Peshawar 25000, Pakistan

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ABSTRACT

In this paper, we consider a transmission model of hepatitis B virus by taking into account media coverage. First, we formulate the model and find the basic reproduction number $\mathcal{R}_0$ by using the next generation matrix method. We show that the disease free equilibrium is locally asymptotically stable for $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$. We also prove that the system is globally asymptotically stable for $\mathcal{R}_0 < 1$. In order to control the spread of this disease in community, we devise an optimal control problem by introducing three control functions, that is, the educational campaign, vaccination and the media coverage. To do this, we solve analytically the control problem with characterization of control variables using the Portraying's Maximum Principle. Finally, we present some numerical illustrations.

1. Introduction

Hepatitis B is a viral infection that attacks the liver and can cause both acute and chronic disease. This virus is transmitted through contact with the blood or other body fluids of an infected person. About 6 million people die every year due to the acute or chronic consequences of hepatitis B. The disease such as SARS, flu and hepatitis B have some distinct properties like as visible symptoms and rapid spread [1]. Communicable diseases spread from one person to another or from an animal to a person. The spread often happens via airborne viruses or bacteria, but hepatitis B spread through blood or other bodily fluid. The terms infectious and contagious are also used to describe communicable disease.

In all health care settings, particularly those in which people are at high risk for exposure to HBV, policies and procedures for HBV control should be developed, reviewed periodically, and evaluated for effectiveness to determine the actions necessary to minimize the risk for transmission of HBV. There are certain measures that can be implemented to control the infection of HBV like administrative measures, environmental controls and media coverage. The role of mass media are greatly acknowledged as a key tool in risk communication [2,3], but the threat of a spectacle for the audience anxiously beneficiaries have been subjected to criticism [4,5]. However, the media reporting role is regarded important in crises, management and perception [4].

In order to understand the dynamics of these control measures several researchers used the theory of mathematical modelling. These model represent various outbreaks and sustained oscillations of emerging and re-emerging infectious diseases [1]. Under the influence of media coverage Liu and Cui [6] presented a mathematical model of an infectious disease

* Corresponding author at: Department of Mathematics, City University of Science and Information Technology, Khyber Pakhtunmkhwa, Peshawar 25000, Pakistan.
E-mail addresses: altafdir@gmail.com, makhan@cusit.edu.pk (M.A. Khan).

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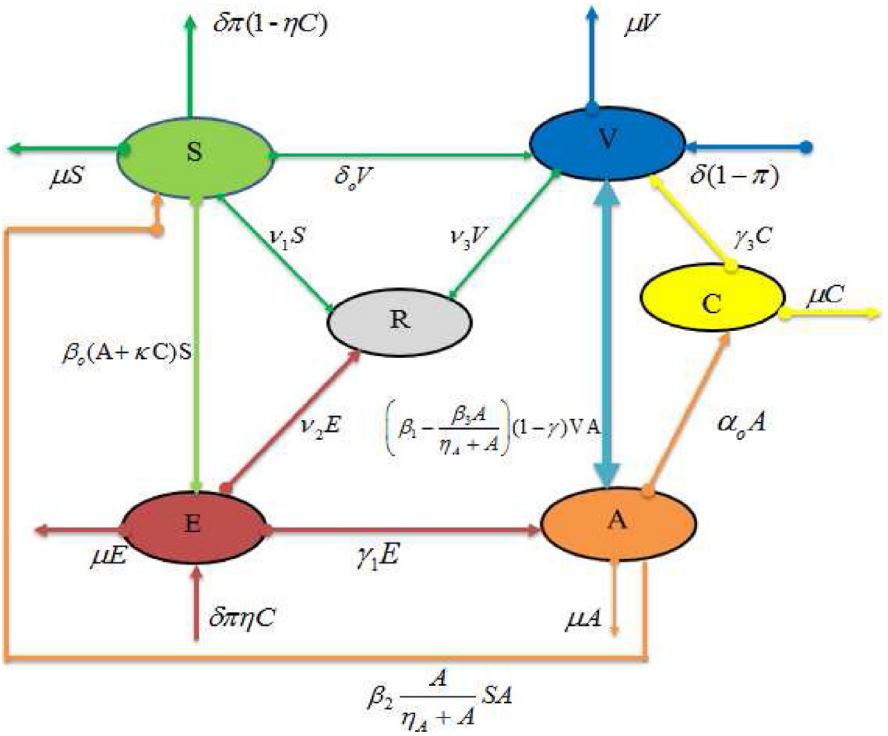

Fig. 1. The flow chart shows the hepatitis B virus transmission.

that give information about the disease spread and control. A pulse and constant vaccination has been incorporated in *SIS* epidemic models by Li and Cui [7]. Cui et al., [8] proved sufficiently strong media impact and shown that with exponential incidence rate alerts every susceptible individuals in a population [1].

Vaccination is a best tool for eliminating the burden of infectious diseases [9]. Despite its benefits on public health it has received a lot of criticism. People often consider it risky for themselves and thus are inclined to refuse it. As an example, we mention the recent rumors about the vaccine campaign of polio in Nigeria that might cause sterility and HIV [10], and the widespread fear that some vaccines might cause autism[11]. Reports show that those individuals who takes vaccine gaining a positive effects on the disease transmission and also the be havior therapy has a great impression on the course of the disease [12,13].

In this work, we develop a compartmental mathematical model of hepatitis B by taking into account the media effect on the community. We study the dynamics of hepatitis B in the presence of available vaccine and the role of media to make awareness of vaccination when the infection is in full swing in the community. We find the basic reproduction number and prove that the disease free equilibrium is stable locally as well as globally. The disease free equilibrium, is both locally and globally unstable when $\mathcal{R}_0 > 1$. In order to control the spread of this infection we use three control variables such as educational campaign, available vaccination and control through media coverage. To do this we use optimal control theory to find our optimal problem. Finally, we solve both problems, with and without control, numerically and illustrate the obtained results.

The structure of the paper is as follows: In Section 2, we formulate our proposed model. Section 3 is devoted to the basic properties of the model. The stability analysis of the model is presented in Section 4. In Section 5, we present the optimal control treatment. The existence of an optimal control problem is discussed in Section 6. We show numerical simulation of both the models in Section 7. Finally, we give some conclusions.

2. Model framework

We divided the total population into five subclasses: susceptible $S(t)$, exposed $E(t)$, acute $A(t)$, carrier $C(t)$, vaccinated individuals $V(t)$ and recovered individuals $R(t)$. This model based on the characteristics of hepatitis B virus transmission with media coverage. The population of susceptible individual is increased by birth or immigration, due to loss of immunity or natural infection. The susceptible move to vaccinated class after vaccination. Similarly the complete transmission in each subclass is represented in the flow chart given in Fig. 1.