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Finite-time boundary control for delay reaction–diffusion systems☆

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ABSTRACT

This paper considers finite-time stabilization and H_∞ performance for delay reaction–diffusion systems by boundary control. First, a full-domain controller is designed and sufficient conditions are obtained to achieve finite-time stability using finite-time stability lemma and Wirtinger's inequality method. Then a boundary controller furnished with sufficient conditions to achieve finite-time stability is presented. When taking into consideration external noise on a delay reaction–diffusion system, finite horizon H_∞ boundary control with a criterion that guarantees the H_∞ performance of delay reaction–diffusion systems is proposed. How to handle Neumann boundary conditions and mixed boundary conditions are discussed. Numerical simulations are carried out to verify the effectiveness of our theoretical results.

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1. Introduction

Reaction–diffusion system characterizes phenomena in fluid dynamics, chemical processes and neural networks in which the state depends on not only time but also spatial position [1–5]. Stability of Reaction–diffusion systems has attracted considerable interests [6–8]. Among these stability results, asymptotical stability is the most studied.

Stability in finite-time is often called for in practical applications, which requests the system state to reach in finite time the original point or a given domain [9–21]. In [16], Bhat and Bernstein provided the mathematical foundation for finite-time stability. From then on, the results were generalized to different kinds of systems and different versions of finite-time stability theorem had been established. In [17], Yin et al. presented the finite-time stability theorem for nonlinear stochastic differential system. Time delay is an important factor that affects the system's properties [22–25]. In [18], a delay differential system was considered and according criterion for finite-time stability of delay systems was given. However, few results have been reported on finite-time stability of delay Reaction–diffusion systems (DRDSs).

As spatial domain is part of the Reaction–diffusion system, this domain induces an important practical control strategy, namely boundary control. Indeed, boundary control has received attention for partial differential systems [26–29]. Krstic and Smyshlyaev have provided a range of boundary controllers based on the backstepping method [30]. Boundary control for

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DRDSs has also been considered [31]. There is however little work done for finite-time stabilization of DRDSs via boundary control.

When a given system is disturbed by external noise, H_∞ control is a strategy of choice to attenuate the disturbance [32–35]. The problem of H_∞ performance for Reaction–diffusion system has also been studied [36,37]. However, few results are on the H_∞ boundary control for DRDSs. How to design boundary controllers to achieve H_∞ performance is of interest in many applications.

Motivated by the above analysis and the problems still to be solved, this study considers the boundary control for DRDSs, focusing on the problems of boundary stabilization and H_∞ boundary control for DRDSs with Neumann boundary conditions. The main difficulty is in the design of finite-time boundary controller. There are few results on the finite-time boundary control for delay Reaction–diffusion systems, and none on systematic approach for the design of finite-time boundary controller. In this paper, first, we study in detail finite-time stability for DRDSs under full-domain control. Then, a boundary controller is designed and sufficient conditions are derived to achieve finite-time stability for DRDSs. When the DRDS is subject to external disturbance, finite horizon H_∞ performance is studied and a robust boundary controller is designed. We propose an effective criterion to ensure the finite horizon H_∞ performance for the disturbed DRDS. The case of mixed boundary conditions is further investigated. Numerical simulations are used to verify our theoretical results.

The main contributions of this paper are listed as follows:

1. A boundary controller is designed for finite-time stabilization of DRDSs. To our knowledge, there is no systematic method to design a finite-time controller for DRDSs and the finite-time boundary controller. Our results can provide a framework for studying of finite-time stabilization for other Reaction–diffusion systems, such as stochastic delay Reaction–diffusion systems.
2. A novel finite horizon H_∞ boundary control is proposed for DRDSs subject to external disturbance that meet H_∞ performance, and a criterion for H_∞ performance is presented.
3. How to handle Neumann boundary conditions and mixed boundary conditions are discussed in the designs of boundary controllers that achieve stabilization and H_∞ performance for DRDSs.

Notation: $\mathbb{R}^n$ is the $n-$dimension Euclidean space. $C[(0,1),\mathbb{R}^n]$ means the set of continuous function $f:(0,1)\to\mathbb{R}^n$. $W^{l,2}([0,L];\mathbb{R}^n)$ is a Sobolev space of absolutely continuous $n-$dimensional vector functions $\omega(x):[0;L]\to\mathbb{R}^n$ with square integrable derivatives $\frac{\mathrm{d}^l\omega(x)}{\mathrm{d}x^l}$ of the order $l\geq 1$. For a constant matrix P, $P\geq 0(\leq 0)$ means that matrix P is a semi-positive (semi-negative) definite matrix.

2. Preliminaries

We consider the following delay Reaction–diffusion system (DRDS)

$$\frac{\partial y(x,t)}{\partial t}=Ay(x,t)+By(x,t-\tau)+C\frac{\partial^2 y(x,t)}{\partial x^2}, x\in(0,1), t>0, \tag{2.1}$$

where $y(x,t)\in\mathbb{R}^n$ is the system state, and x, t are the spatial variable and time variable, respectively and τ is the time delay. System coefficients A, B and C are constant matrices with C taken as positive definite. The following initial value is imposed on system (2.1)

$$y(x,s)=\phi(x,s), s\in[-\tau,0], \tag{2.2}$$

We give the definition of finite-time stability for delay Reaction–diffusion system (2.1). This definition is inspired by the one in [16].

Definition 2.1 [16]**.** The zero solution of delay Reaction–diffusion system (2.1) is finite-time convergent if there exist an open neighborhood $U\subset C[(0,1),\mathbb{R}^n]$ of the origin and a functional $t^*(\phi)$: $U\backslash\{0\}\to(0,\infty)$, such that for all $\phi\in U$, the solution $y(x, t, \phi)$ of system (2.1) with the initial value $\phi(x, s)$ satisfies $y(x, t, \phi)\in U\backslash\{0\}$ for $t\in[0, t^*(\phi)]$ and $\lim_{t\to t^*(\phi)} y(x,t,\phi)=0$. We call $t^*(\phi)$ the settling time with the initial value ϕ. The zero solution of delay Reaction–diffusion system (2.1) is said to be finite-time stable, if it is Lyapunov stable and finite-time convergent. If the zero solution of system (2.1) is finite-time stable, we also call delay Reaction–diffusion system (2.1) is finite-time stable.

The finite-time stability theorem in [16] can be generalized to the following lemma for delay Reaction–diffusion systems. Since the proof is similar to that in [16], we omit it here for brevity. This lemma is the foundation of this study.

Lemma 2.2 (Finite-time stability [16])**.** *Suppose that function $V(y(\cdot, t))$: $[0, \infty)\to[0, \infty)$ is differentiable (the derivative of $V(y(\cdot, t))$ at 0 is in fact its right derivative) and*

$$\frac{\mathrm{d}V(y(\cdot,t))}{\mathrm{d}t}\leq -kV^{\beta}(y(\cdot,t)), \tag{2.3}$$

where $k>0$ and $0<\beta<1$. Then $V(y(\cdot, t))$ will reach zero at finite time $t^*\leq\frac{V^{1-\beta}(y(\cdot,0))}{k(1-\beta)}$ and $V(y(\cdot,t))=0$ for all $t\geq t^*$.

The following lemma plays an important role in the analysis.

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