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An iterative algorithm for the least Frobenius norm least squares solution of a class of generalized coupled Sylvester-transpose linear matrix equations☆

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ABSTRACT

The iterative algorithm of a class of generalized coupled Sylvester-transpose matrix equations is presented. We prove that if the system is consistent, a solution can be obtained within finite iterative steps in the absence of round-off errors for any initial matrices; if the system is inconsistent, the least squares solution can be obtained within finite iterative steps in the absence of round-off errors. Furthermore, we provide a method for choosing the initial matrices to obtain the least Frobenius norm least squares solution of the problem. Finally, numerical examples are presented to demonstrate that the algorithm is efficient.

1. Introduction

Matrix equations appear frequently in many areas of applied mathematics and play important roles in many applications, such as control theory, system theory [19,20]. For example, in stability analysis of linear jump systems with Markovian transitions, the following matrix equations are typical coupled Lyapunov matrix equations

$$A_i^T + P_iA_i + Q_i + \sum_{j=1}^{n} \pi_{ij}P_j = 0, \quad i = 1, 2, \dots, n,$$

where Q_i are positive definite matrices, π_{ij} are known transition probabilities and P_j are the unknown matrices [6,43]. The second order linear system

$$A_2\ddot{x} + A_1\dot{x} + A_1\dot{x} + B_0u = 0$$

has wide applications in vibration and structural analysis, robotics control and spacecraft control [39,57]. All kinds of publications have studied how to solve different types of matrix equations [14,15]. Traditionally, linear matrix equations can be converted into their equivalent forms by using the Kronecker product. However, in order to solve the equivalent forms, the

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inversion of the associated large matrix need be involved, which leads to computational difficulty because excessive computer memory is required. With the increase of the sizes of the related matrices, the iterative methods have replaced the direct methods and become the main strategy for solving the matrix equations [5,18,19].

Based on the conjugate gradient algorithm, there are several iterative algorithms for solving the (coupled) linear matrix equations [4,9–13,15,16,35–37,48,50,51,54]. Bai [1] established the Hermitian and skew-Hermitian splitting iteration methods for continuous Sylvester matrix equations. Beik and Salkuyeh [5] derived the global Krylov subspace methods for solving general coupled matrix equations. Deng et al. [17] constructed orthogonal direction methods for Hermitian minimum norm solutions of two consistent matrix equations. Zhou et al. [56] obtained the solutions of a family of matrix equations by using the Kronecker matrix polynomials. Ding et al. [18] constructed the iterative method for finding the solutions of the generalized Sylvester matrix equations by using the hierarchical identification principle. The generalized conjugate direction algorithm for solving the general coupled matrix equations over symmetric matrices was derived by Hajarian [27]. The matrix forms of CGS, GPBiCG, QMRCGSTAB, BiCOR, Bi-CGSTAB, CORS, BiCG, Bi-CR and CGLS algorithms were given to solve linear matrix equations [25,28,30–34].

The Kalman–Yakubovich-transpose matrix equation $X - AX^TB = C$, the generalized Yakubovich-transpose matrix equation $X - AX^TB = CY$, the nonhomogeneous Yakubovich-transpose matrix equation $X - AX^TB = CY + R$ and the Sylvester-transpose matrix equation $AX + X^TB = C$ play very important roles in many fields [3,55]. For example, the general Lyapunov-transpose and the Kalman–Yakubovich-transpose matrix equations appear in the Luenberger-type observer design [47], pole/eigenstructure assignment design [40] and robust fault detection [7]. The generalized Yakubovich-transpose matrix equation is encountered in second order or higher order linear systems [21,46]. The Sylvester-transpose matrix equation is related to the eigenstructure assignment [23], observer design [8], control of system with input constraint [22], and fault detection [24]. In [42,53], the following linear matrix equations

$$\sum_{i=1}^{r} A_i X B_i + \sum_{j=1}^{s} C_j X^T D_j = E, \tag{1.1}$$

where A_i, B_i, C_j, D_j, $i = 1, \ldots, r,\ j = 1, \ldots, s$, and E were some known constant matrices of appropriate dimensions and X was a matrix to be determined, was considered. The special case of Eq. (1.1), that is

$$\sum_{i=1}^{k} \left(A_i X B_i + C_i X^T D_i \right) = E \tag{1.2}$$

was considered by Hajarian. He established matrix iterative methods [29] and QMRCGSTAB algorithm [34] for solving Eq. (1.2). The special case of Eq. (1.1) $AXB + CX^TD = E$ was considered by Wang et al. [49]. Best approximate solution of matrix equation $AXB + CXD = E$ was studied in [41]. In [52], the gradient-based iterative algorithms were established for $AXB + CX^TD = F$ which is also a special case of Eq. (1.1). A more special case of Eq. (1.1), namely, the matrix equation $AX + X^TC = B$, was investigated by Piao et al. [44]. Using the Moore–Penrose generalized inverse, some necessary and sufficient conditions for the existence of the solution and the expressions of the matrix equation $AX + X^TC = B$ were obtained in [44].

Moreover, the following generalized coupled Sylvester-transpose matrix equations

$$\sum_{\eta=1}^{p} \left(A_{i\eta} X_\eta B_{i\eta} + C_{i\eta} X_\eta^T D_{i\eta} \right) = F_i, \quad i = 1, 2, \ldots, N \tag{1.3}$$

was considered by Song et al. [45]. They obtained the least Frobenius norm solution group and the optimal approximation solution group of system (1.3). Beik and Salkuyeh [4] also considered the coupled Sylvester-transpose Eq. (1.3) over generalized centro-symmetric matrices. As a special case of Eq. (1.3), Dehghan and Hajarian researched the generalized centro-symmetric and least squares generalized centro-symmetric solutions of the matrix equations $AYB + CY^TD = E$ [15]. Baksalary and Kala [2] studied the matrix equation $AXB + CYD = E$. Hajarian [26] established the new finite algorithm for solving the generalized nonhomogeneous Yakubovich-transpose matrix equation $AXB + CX^TD + EYF = R$.

In [54], Xie et al. considered the following generalized coupled Sylvester-transpose linear matrix equations

$$\begin{cases} AXB + CY^TD = S_1, \\ EX^TF + GYH = S_2, \end{cases} \tag{1.4}$$

where $A, E \in R^{p\times n}$, $C, G \in R^{p\times m}$, $B, F \in R^{n\times q}$, $D, H \in R^{m\times q}$, $S_1, S_2 \in R^{p\times q}$ are given constant matrices, and $X \in R^{n\times n}$, $Y \in R^{m\times m}$ are unknown matrices to be determined. This kind of matrix equation can be used in future works of control and system theory. Xie et al. [54] proved that the solution can be obtained within finite iteration steps in the absence of round-off errors for any initial given reflexive or anti-reflexive matrix as system (1.4) is consistent. However, as system (1.4) is inconsistent, how to obtain the least squares solution and the least Frobenius norm least squares solution is still open.

In this paper, the problems will be tackled in a new way. Inspired by the previous works, we propose a modified conjugate gradient method to solve system (1.4). We consider two cases. When system (1.4) is consistent, we verify that a solution (X^*, Y^*) can be obtained within finite iteration steps in the absence of round-off errors for any initial matrices. When system (1.4) is inconsistent, we prove that the least squares solution of system (1.4) can be obtained within finite iteration steps