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Estimation of biophysical parameters in a neuron model under random fluctuations

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ABSTRACT

In this paper, an attempt has been made to estimate the biophysical parameters in an improved version of Morris–Lecar (M–L) neuron model in a noisy environment. To observe the influence of noisy stimulation in estimation procedure, a Gaussian white noise has been added to the membrane voltage of the model system. Estimation of the parameters has been investigated by a proposed algorithm. The denoising technique (local projection method) has been applied to reduce the influence of noisy stimuli and the effectiveness of the method is reported. The proposed scheme performs well for an excitable neuron model and provides good estimates between the estimated parameters and the actual values in a reasonable way. This approach can be used for parameter estimation for other nonlinear dynamical systems.

1. Introduction

Estimation of parameters in biological excitable systems given discrete time measurement data is a challenging problem addressed by many researchers in mathematical/computational neuroscience. Existing methods for estimation of parameters in dynamical systems involve multiple-shooting methods, hybrid technique, least-square scheme and stochastic methods etc. [1–3]. In neuronal system, neurons can generate multiple modes in electrical activities, and neurons can keep synchronization and anti-synchronization. It provides an another effective scheme to estimate the unknown parameters in neurons [4,5]. Chou and Voit [6] have studied the problem associated with the parameter estimation in biological systems. It is interesting to identify the parameters in the model that shows good agreement with the observed data. Based on Lyapunov stability criterion and adaptive synchronization technique, optimization design of adaptive controllers and parameter observers have been studied with controllable gain coefficient using different hyperchaotic systems [7]. Now, it is very crucial task to develop effective and reasonable mathematical tools for deep research studies in real world applications of neural computation. In neural dynamics, one important question is that what type of neural model can be considered to study the dynamical behavior of the neuron. A number of research articles have been proposed involving the parameter estimation of neural models based on dynamical system theory from various perspectives. The methods were examined in noisy measurements as well as in the absence of noise like stimuli.

Tabak et al. [8] proposed two techniques (time domain and frequency domain approaches) to estimate the parameters of single neuron models and observed that it can estimate the active and passive dynamical properties with multiple

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active conductances. However, the time domain approach is slower and more liable to estimation errors than the frequency domain technique. There are some research articles concerning the parameter estimation of excitable models using various techniques. Maximum likelihood method to estimate the parameters was developed by Pillow et al. [9] in a one dimensional integrate-and-fire model under noisy environment. The stochastic Ornstein–Uhlenbeck [10] models were studied to estimate the input parameters based on the firing regime of the process. The system variables and associated parameters of the neuron models such as FitzHugh–Nagumo (FHN), Morris–Lecar (M–L) and Hindmarsh–Rose (H–R) models can be reconstructed. Later a stochastic diffusive leaky integrate-and-fire model for a slowly fluctuating signal was examined by Picchini et al. [11] to estimate the biophysical parameters. It was shown and analytically derived that the firing rate and CV uniquely determine the parameters of the perfect, leaky and quadratic integrate and fire neuron models [12]. A method has been proposed by Tyukin et al. [13] from in vitro experiments, how to recover the system variables and parameters of canonical neural oscillators. Schumann-Bischoff and Parlitz [14] presented a method based on nonlinear optimization which is efficient in estimating the variables and parameters of a system of ordinary differential equations (ODEs). To fit appropriate model parameters for membrane voltage recordings, it is often a difficult task to compute manually. The model parameters of a FHN system were estimated under the presence of noise [15,16]. Now, a reconstruction method was proposed by Odom and Borisyuk [17] for the estimation of noisy synaptic conductances in a neural model. Recently, Ditlevsen and Samson [18] proposed a method for parameter estimation using a maximum likelihood technique in a stochastic two dimensional M–L model. A sequential Monte Carlo particle filter algorithm [18] has been designed to impute the unobserved coordinate and estimate the required parameters in the stochastic model using the Expectation–Maximization algorithm. To deal with the parameter estimation, Leander et al. [19] considered a stochastic differential equation for continuous time dynamical system with discrete time measurements. In our recent work, we proposed a method on parameter estimation using method of moments in a spiking–bursting Hindmarsh–Rose (H–R) neural model under noisy measurements [20].

The dynamical behavior of the modified 3D M–L model [21–24] is interesting to analyze the excitable neural behavior. It is a modified version of the well-known M–L model [25]. The biophysical behavior and efficient computational performances of M–L model are important in practical applications for neural computation. M–L system has significant contribution in information processing and temporal coding. This 3D model is different from the previous works on M–L, FHN and other neural models which have only spiking features. We are interested to study the estimation of parameters in a noisy dynamical model exhibiting various types of bursting at certain parameter regime. The bursting in neural models was proposed by Rinzel [26] and later extensively studied in [21,22]. The previous models mentioned above were studied with a constant injected current. The 3D M–L system presents tonic bursting if the extra equation is added i.e., the applied current is considered as a slow variable. It is considered as a fast-slow system. This type of bursting is observed in regular spiking excitatory neurons, low threshold spiking neurons [27,28], cat neocortex chattering neurons [29] and so on. Modified M–L model [21,22] produces different neuro-computational features which occur in real neurons corresponding to appropriate choices of various set of parameters. However, the parameters of the M–L model cannot be estimated in a straight forward manner due to its nonlinearity and coupled complex feature. It is very interesting to study transmission and information processing in single M–L neurons. These types of bursting are largely used in the simulation of cortical neural networks [21,22].

Brain is a noisy complex nonlinear dynamical network system and its functional behavior produces excitatory or chaotic type activities. Usually, the noisy behavior arises from various sources such as release of neurotransmitters in the synapses, the on-off conditions of ion-channels across the membranes, random synaptic inputs from the adjacent neurons of a neural network and so on [30]. The biophysical systems can be considered as noisy system and noise can emerge from many factors [31]. How noise affects neuronal dynamics, or more generally excitable dynamics, has been studied in the references [32,33]. Recently, an exact theory for the noise-driven activation of excitable neurons (using FitzHugh–Nagumo equations) and large assemblies of type II excitable units have been developed in activation process in excitable systems with multiple noise sources [34,35]. The effects of different network topologies on noise-induced dynamics have also been studied in effects of small-world connectivity on noise-induced temporal and spatial order in neural media with FitzHugh–Nagumo local dynamics [36]. The phenomenon of spatial coherence resonance was studied with FitzHugh-Nagumo local dynamics by Perc [37]. The different network topologies on the noise-induced pattern formation were also studied using the FitzHugh–Nagumo excitable media [38]. It was also investigated the effects of spatiotemporal additive noise in conjunction with subthreshold travelling waves on the spatial dynamics of excitable media [39]. These are some well-known articles regarding the noise induced excitable systems. The impact of noise can be controlled in a systematic way to a certain extent. We have proposed plausible techniques for parameter estimation and investigate the effectiveness of the method in the above mentioned neuron model.

Unlike the former reviewed methods, the main contribution of this work is to propose an adaptive scheme for estimation of biophysical parameters of a nonlinear 3D M–L model during the dynamics of oscillatory patterns. The estimation procedure is based on the method of least square to deal with the distribution of the biophysical parameters [40,41]. Parameter estimation in the presence of noise may lead to misleading results. To effectively estimate the parameters of the model, it is necessary to reduce the impact of noise [42,43]. The present study addresses a denoising technique based on local projection [44–46] which depends on proper estimation of lag parameter and embedding dimension [47,48].

The paper is organized as follows: Section 2 presents the formulation of a 3D modified version of the M–L model. The formulation of re-parametrization and the method of estimation have also been described. In Section 3, numerical results