Applied Mathematics and Computation 328 (2018) 171–188

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Numerical solution of high-order Volterra–Fredholm integro-differential equations by using Legendre collocation method

N. Rohaninasab, K. Maleknejad*, R. Ezzati

Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran

ARTICLE INFO

MSC:
45J05
65R20
34K06
34K28

Keywords:
Neutral and high-order Volterra–Fredholm integro-differential equation
Legendre-spectral method
Guass quadrature formula
Convergence analysis

ABSTRACT

The main purpose of this paper is to use the Legendre collocation spectral method for solving the high-order linear Volterra–Fredholm integro-differential equations under the mixed conditions. Avoiding integration of both sides of the equation, we expressed mixed conditions as equivalent integral equations, by adding the neutral term to the equation. Error analysis for approximate solution and approximate derivatives up to order k of the solution is obtained in both L^2 norm and L^∞ norm. To illustrate the accuracy of the spectral method, some numerical examples are presented.

© 2018 Elsevier Inc. All rights reserved.

1. Introduction

Integro-differential equations provide an important tool for modeling physical phenomena in various fields of science and engineering. Many problems in Elecrtomagnetics [1–10], Finance [11], Chemistry, Astronomy, Biology, Economics, Potential theory, Electrostatics [12–16], Mechanics [17–19] lead to these equations. The numerical solutions of such equations have been studied by many authors. A Taylor expansion approach for the nonlinear case has been presented by Yalçinbas [20] and also by Yalçinbas and Sezer [21] for the linear case. These two works are based on differentiating both sides of the integral equation n times, substituting the Taylor series for the unknown function in the resulting equation and the matrix relations. Shahmorad [22] considered Oritz and Samara's operational approach to the Tau method for the differential part of the general case of Volterra–Fredholm Integro-Differential Equation (VFIDE). Akyuz-Dascioglu [23] applied Chebyshev polynomials to transform VFIDE and the conditions into the matrix equations. Babolian et al. [24] used Block Pulse Functions and its operational matrix of integration to convert a nonlinear of VFIDE into a nonlinear system of algebraic equations. Biazar and Eslami [25] proposed He's Homotopy perturbation method for the nonlinear case of VFIDE. In [26], the Volterra integro-differential equation up to order k is studied with initial conditions and error analysis is presented in L^2 norm and L^∞ norm. Here, we generalize the method presented in [26], to solve Volterra–Fredholm integro-differential equation of order k under mixed conditions. The obtained error analysis is similar to [26]. Now, this paper considers the following high-order VFIDE

$$\sum_{k=0}^{m} p_k y^{(k)}(x) = f(x) + \lambda_1 \int_a^x K_1(x,s)y(s)ds + \lambda_2 \int_a^b K_2(x,s)y(s)ds, \tag{1}$$

* Corresponding author.
E-mail address: maleknejad@iust.ac.ir (K. Maleknejad).

https://doi.org/10.1016/j.amc.2018.01.032
0096-3003/© 2018 Elsevier Inc. All rights reserved.

with the mixed conditions

$$\sum_{k=0}^{m-1} a_{jk}y^{(k)}(a) + b_{jk}y^{(k)}(b) + c_{jk}y^{(k)}(c) = \mu_j, \quad 0 \le j \le m-1, \quad a \le c \le b, \tag{2}$$

where $y(x)$ is an unknown function, $K_1(x, s)$, $K_2(x, s)$ and $f(x)$ are analytical functions that have suitable derivatives on $D = \{(x, s) : a \le s \le x \le b\}$ and $[a, b]$, respectively. Also p_k, λ_1, λ_2, a_{jk}, b_{jk}, c_{jk} and $\mu_j (j = 0, \dots, m-1)$ are constants.

In order to use the Guass-quadrature rules, we will transfer the problem (1) and (2) to an equivalent problem in $[-1, 1]$. We use the change of variable

$$x = \frac{b-a}{2}\tau + \frac{b+a}{2}, \quad \tau \in [-1, 1].$$

To rewrite Eqs. (1) and (2) as follows:

$$\sum_{k=0}^{m} q_k u^{(k)}(\tau) = g(\tau) + \lambda_1 \int_a^{\frac{b-a}{2}\tau+\frac{b+a}{2}} K_1\left(\frac{b-a}{2}\tau + \frac{b+a}{2}, s\right)y(s)ds$$
$$+ \lambda_2 \int_a^b K_2\left(\frac{b-a}{2}\tau + \frac{b+a}{2}, s\right)y(s)ds, \tag{3}$$

$$\sum_{k=0}^{m-1} \left(\frac{2}{b-a}\right)^k \left[a_{jk}u^{(k)}(-1) + b_{jk}u^{(k)}(1) + c_{jk}u^{(k)}(\rho)\right] = \mu_j, \quad 0 \le j \le m-1, \tag{4}$$

in which

$$u(\tau) = y\left(\frac{b-a}{2}\tau + \frac{b+a}{2}\right), \quad g(\tau) = f\left(\frac{b-a}{2}\tau + \frac{b+a}{2}\right),$$
$$q_k = \left(\frac{2}{b-a}\right)^k p_k, \quad \rho = \frac{2c-(b+a)}{b-a}.$$

Now, to transfer the integration interval $[a, \frac{b-a}{2}\tau + \frac{b+a}{2}]$ to the $[-1, \tau]$, we make a linear transformation

$$s = \frac{b-a}{2}\eta + \frac{b+a}{2}, \quad \eta \in [-1, \tau].$$

Then Eq. (3) becomes

$$\sum_{k=0}^{m} q_k u^{(k)}(\tau) = g(\tau) + \delta_1 \int_{-1}^{\tau} \widehat{K}_1(\tau, \eta)u(\eta)d\eta + \delta_2 \int_{-1}^{1} \widehat{K}_2(\tau, \theta)u(\theta)d\theta, \tag{5}$$

where

$\widehat{K}_1(\tau, \eta) = K_1\left(\frac{b-a}{2}\tau + \frac{b+a}{2}, \frac{b-a}{2}\eta + \frac{b+a}{2}\right), \quad \delta_1 = \frac{b-a}{2}\lambda_1, \quad \delta_2 = \frac{b-a}{2}\lambda_2,$

$\widehat{K}_2(\tau, \theta) = K_2\left(\frac{b-a}{2}\tau + \frac{b+a}{2}, \frac{b-a}{2}\theta + \frac{b+a}{2}\right).$

2. Legendre-colloction method

In this section, we describe the process of solving the problems of (4) and (5) on domain $[-1, 1]$. A Sturm–Liouville problem is an eigenvalue problem of the form

$$-(pu')' + qu = \lambda wu \quad \textit{in the interval } (-1, 1). \tag{6}$$

The Legendre polynomials $L_k(x), k = 0, 1, \dots,$ are the eigenfunctions of the singular Sturm–Liouville problem

$$((1-x^2)L'_k(x))' + k(k+1)L_k(x) = 0, \tag{7}$$

which is (6) with $p(x) = 1 - x^2$, $q(x) = 0$ and $w(x) = 1$. The set of $\{L_k(x)\}_{k=0}^{\infty}$ forms a complete L^2 orthogonal system in $(-1, 1)$. Let the collocation points be $(N+1)$ Legendre–Gauss, or Gauss–Radau, or Gauss–Lobatto points, $\{x_i\}_{i=0}^{N}$ with the corresponding weights, $\{w_i\}_{i=0}^{N}$. Similarly [26], we restate the mixed conditions (4) as an equivalent integral equations. If we replace

$$u^{(k)}(\tau) = u^{(k)}(-1) + \int_{-1}^{\tau} u^{(k+1)}(\eta)d(\eta), \tag{8}$$

$$u^{(k)}(\tau) = u^{(k)}(1) - \int_{\tau}^{1} u^{(k+1)}(\eta)d(\eta), \tag{9}$$

$$u^{(k)}(\tau) = u^{(k)}(\rho) + \int_{\rho}^{\tau} u^{(k+1)}(\eta)d(\eta), \tag{10}$$

Download English Version:

https://daneshyari.com/en/article/8901094

Download Persian Version:

https://daneshyari.com/article/8901094

Daneshyari.com